More Than Rules

College Transition Math Teaching
for GED Graduates at The City University of New York

written by

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Introduction

Much has been written about the high percentage of high school and GED graduates who enter community colleges needing remedial coursework and the low rates of retention and graduation for these students. Reports on how to improve outcomes for underprepared students often focus on the merits of adopting specific program components such as learning communities, computer-assisted instruction, accelerated learning, supplemental instruction, career-based curricula, intensive advisement, or faculty inquiry groups. Certainly, many of these can be useful features of a high-quality transition or remedial program. Unfortunately, though, too little attention is given to exactly how instructors teach students in these classrooms. There is an urgent need to re-examine the ways we teach underprepared students entering college. Re-focusing attention on pedagogy must also cause us to re-think how we approach content, assessment, curricula, staff development, student placement, and research.

This paper describes how the College Transition Program (CTP) has attempted to strengthen GED graduates' transition into The City University of New York (CUNY) through a semester of reading, writing, mathematics, and academic advisement. More precisely, this paper focuses on math teaching and learning in CTP.

CTP has worked almost exclusively with GED graduates, but we believe the early results will be interesting to a variety of programs working with students who enter college underprepared in math including GED programs more widely, college remedial math departments, and high schools. This paper is for instructors and administrators who work in these settings as well as for researchers, policy makers, and funders. At times, some technical math teaching language may be used but the bulk of these instances are limited to the footnotes and appendices. Much of this paper should be readable by a wide audience.

This paper opens with a description of basic skills testing for students entering Associate's Degree programs at CUNY colleges. Student performance on college placement exams and especially the math exams helps to clarify how often and in which subjects students place into remediation at CUNY.

The second section focuses on the poor alignment in math content between the GED and the COMPASS exams used for placement at CUNY. This misalignment along with the reality that a large number of GED graduates have significant math weaknesses help to explain why the vast majority of GED graduates fail the COMPASS exams.

Remedial math classes await students who fail math placement exams, and data are provided in the third section of this paper that detail student outcomes in these courses at CUNY. Low pass rates in remedial math courses are common in community colleges across the country, and this helped to convince us that we should try a fresh approach to math instruction in CTP.

The fourth section gives a brief history of CTP with a special focus on the significant changes we have made in our academic and advisement models over time in response to the needs of our students. It has been a challenge to write this paper precisely because we have refashioned the program quite dramatically over the early pilot semesters, making CTP a moving target. Still, we did provide a consistent, intensive model of instruction and advisement for three cohorts across the fall 2008 and spring 2009 semesters and so the academic and advisement models as well as the student outcomes over that period are highlighted here.

The CUNY math placement exams are high-stakes tests and section five describes what we have learned about them. It has been disappointing to discover that very little information is available on the content that is valued in the exams, and when students complete the exams, we receive little useful information about what mathematics they can do or where they need to improve.

Before focusing on the CTP approach to math content, pedagogy, curriculum, and staff development, section six describes some common teaching and learning practices in college remedial math programs.

Despite having little good information on the content of the CUNY math placement exams, we had to decide what content to teach CTP students. One of the most important decisions we have made is to break from the common practice of covering long lists of topics at a rapid pace. We decided from the very first CTP semester that we would teach in ways that develop deep understanding in our students, even when this limits the number of topics we may study. These and other decisions we made about math content are described in section seven.

Section eight is titled *Math Teaching and Learning in the College Transition Program* and is perhaps the most important in this document because it details our pedagogical philosophy in conjunction with mathematical examples in the appendices for those who wish to review them. An effort is made here to draw linkages between CTP math teaching and recommended practices from a number of research and standards documents.

An important early finding is that we may be demonstrating that “less is more.” CTP students and instructors who do careful work over a narrower set of math topics than is customary in remedial math classes have shown impressive gains in their math ability as measured on CTP assessments, in their confidence and persistence, and on the CUNY math placement tests when compared with their GED graduate peers.

The CTP math curriculum is a “living” document that undergoes revision by the math instructor team each semester and is the backbone of our pedagogical unity across CTP sites. The process of developing and using the curriculum is outlined in section nine. The related work of identifying, inducting, and training a team of skilled math instructors is described in section ten.

Sections eleven and twelve use what we have learned building CTP math to inform a series of recommendations for GED programs and for college remedial math programs. These recommendations focus on content, pedagogy, intensity, curricula, staff development, research, and student placement.

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Gayle Cooper-Shpirt, Hilary Sideris, and Moira Taylor have worked the closest with me in coordinating CTP since its inception. I am very grateful for their patient, skilled, serious approach to this work. Other CUNY central office staff who have played important roles include Tracy Meade, Daniel Voloch, Eric Hofmann, Amy Perlow, David Crook, Joe Schneider, Drew Allen, Kate Brandt, Shirley Miller, Ramon Tercero, Delphine Julian, and Sam Seifnourian.

Many others have given a great deal of time and effort to help make CTP successful, especially at the campuses where CTP classes have taken place. These include Mae Dick, Charles Perkins, Mark Trushkowsky, Ida Heyman, Sarah Eisenstein, Serge Shea, Kieran O'Hare, Marzena Bugaj, Jane MacKillop, Amy Dalsimer, Linda Chin, Mimi Blaber, David Housel, Maggie Arismendi, Paul Arcario, Bret Eynon, Judit Torok, Joan Manes, Denise Deagan, Solange Farina, Nicole Tavares, Lashallah Osborne, Wayne Carey, Irma Lance, Carlo Baldi, Zenobia Johnson, Blanche Kellawon, Donna Grant, Paul Wasserman, Azi Ellowitch, Bernie Connaughton, Mike Dooley, Delana Radameron, Alexis Morales, Frannie Rosensen, Cheryl Georges, and Laura Zan.

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Basic Skills, Math Proficiency, and Retention at The City University of New York

GED graduates entering Associate's Degree programs at CUNY generally take basic skills exams in reading, writing, and mathematics as a part of enrollment. Failing any one of these exams typically means a student must take and pass a remedial course in that subject before re-testing.[1] Students must pass all three of these exams (or earn exemptions) before they are allowed to take courses they need to ultimately earn an Associate's Degree.

Students may be declared "CUNY exempt" and bypass one or more of the placement exams if they previously earned certain minimum scores on the New York State Regents, SAT, or ACT high school exams. The following data show the rates at which GED and high school graduates earned exemptions in the three basic skills areas.

Rate that GED and High School Graduates Earned Basic Skills Exemptions Based on Regents, SAT, or ACT High School Exam Scores[2]

First-Time Freshmen in Associate's Degree Programs Entering Fall 2008

	GED Graduates	NYC Public H.S. Graduates
Reading	4.1%	41.7%
Writing	4.1%	41.0%
Math	3.2%	39.3%

This data shows that the vast majority of GED graduates entering Associate's Degree programs in the fall 2008 semester needed to take the CUNY placement exams because they did not earn exemptions. The exemption rate appears dramatically higher for high school graduates in all three areas, but this is somewhat misleading. Regents, SAT, and ACT exemptions are recognized for reading and writing at all six CUNY community colleges. For at least two colleges, however, they are no longer recognized for mathematics and so all entering students at those campuses must take and pass the math placement exams in order to bypass math remediation. Different standards have led to the confusing situation where students who are officially "CUNY exempt" in math and who would be placed into credit-based math classes at some colleges may be placed in remedial math classes at other colleges.

As was shown above, virtually all GED graduates and a majority of high school graduates entering CUNY must take the math placement exams. The exams are commonly known as "COMPASS Math" and are a product of ACT, Inc.[3] The exams include up to four parts, but the first two (pre-algebra and algebra) are the critical ones

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[1] In some cases, CUNY colleges offer students a second chance to pass a placement exam and avoid a remedial course if they attend a free compressed course in advance of their first semester.

[2] This data was provided by CUNY Collaborative Programs Research and Evaluation. For GED graduates, $n = 1{,}312$ and for high school graduates, $n = 11{,}180$. Data are for students who applied through the central CUNY application system. These figures do not include students who were accepted later through "direct admission". Each year, CUNY community colleges "direct admit" a significant number of students who miss central application deadlines.

[3] At CUNY campuses, the basic skills exams go by many names, including the "CUNY Placement Exams", "Freshmen Skills Tests", "CUNY Assessment Tests", and "ACT Tests". All of the tests are products of ACT, Inc. In this paper, we

students must pass to avoid remedial classes. Normally, a student who fails both parts will need to take and pass two remedial math courses. A student who only fails the second (algebra) exam will need to take and pass one remedial algebra course. A student who passes both parts is considered proficient in math and is given additional parts to determine appropriate placement in a credit-bearing math course.[4]

The following data from the fall 2008 semester show how entering students fared on the CUNY placement exams.

Pass Rates on Initial Placement Exams for GED Graduates and Non-Exempt NYC High School Graduates Who Tested at CUNY[5]

First-Time Freshmen in Associate's Degree Programs Entering Fall 2008

	GED Graduates	Non-Exempt NYC Public H.S. Graduates
Pass rate for reading	70.1%	47.0%
Pass rate for writing	25.9%	21.1%
Pass rate for math part one (pre-algebra)	59.3%	41.3%
Pass rate for math part two (algebra)	14.0%	14.3%
Rate passing in all areas	3.8%	1.5%

Large numbers of students fail both math exams, but the algebra exam is clearly the biggest hurdle faced by GED and non-exempt high school graduates. As was the case in the exemption data, though, individual college practices vary and as a result these rates actually overstate entering students' success on the math exams.[6]

Having large numbers of entering students fail math placement exams is not unique to CUNY colleges. In one sample of 46,000 students entering 27 U.S. colleges, more than 70% needed remedial math instruction.[7] Truly, this is a national problem at community colleges, and holds whether students take the COMPASS math exams used at CUNY, the ACCUPLACER, or another testing product.

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refer to them as the CUNY placement exams in general and the COMPASS math exams in particular. These tests are unrelated to the CUNY Proficiency Exam (CPE), which is given to Associate's Degree students near graduation and Bachelor's Degree students after approximately 60 credits have been earned.

[4] At some CUNY community colleges, students entering STEM majors may pass two COMPASS parts but still be required to enroll in a zero-credit math course if the college determines they are not prepared for the math demands of their major.

[5] This data was provided by CUNY Collaborative Programs Research and Evaluation. Data are for non-exempt testers who applied through the central application system and not those who applied through direct admission. For GED graduates, $n = 1{,}230$ and for high school graduates, $n = 6{,}469$. Data is for the fall 2008 semester (and not earlier) because it was in fall 2008 that entering students faced new, higher minimum passing scores on COMPASS math. Predictably, pass rates were higher in earlier semesters when students could pass with lower scores. In the fall 2007 semester, for example, 72% of entering GED graduates passed math part one and 23% passed math part two. The results for public high school graduates appear worse than for GED graduates, but it is important to remember that, roughly speaking, the strongest one-third of public high school graduates are not included here because they were declared exempt from the exams.

[6] The rates assume that students will pass math part one or two when they earn a COMPASS scaled score of 30. These are the official CUNY passing scores for all community colleges. However, two of the community colleges have increased the minimum passing scores at their own campuses—in one case requiring scores of 40 and 38 and in a second case requiring scores of 30 and 50 for students entering certain courses of study. Data that incorporates these different standards is not available, but higher minimum scores likely mean the actual pass rates in math are lower than the ones shown above.

[7] *Accelerating Remedial Math Education: How Institutional Innovation and State Policy Interact*, An Achieving the Dream Policy Brief by Radha Roy Biswas for Jobs for the Future, 2007, page 1.

Why the COMPASS Math Exams Are Challenging for GED Graduates

Math does not appear as a problem for the first time when a GED graduate reaches CUNY. Many adult students have great difficulty passing the GED math subject test. The following data demonstrates that students fail to reach the minimum passing score on the math test more often than in any other subject.

Failure Rate for GED Testers by Subject Test, 2007[8]		
Subject Test	Rate that New York State Testers Failed to Reach Minimum Score	Rate that U.S. Testers Failed to Reach Minimum Score
Math	26.4%	18.8%
Writing	17.8%	9.2%
Science	12.8%	7.6%
Social Studies	9.0%	6.2%
Reading	7.4%	4.4%

The national data shows that students fail the GED math test more than twice as often as the writing and science tests and more than three times as often as the social studies and reading tests. New York State students have a higher math failure rate than all other states except Mississippi and the District of Columbia. Note also that these figures are for testers who in many cases have been sent to take the GED only after they have studied in an adult literacy program for months or years before demonstrating a reasonable likelihood of passing.

Students who are studying to take the GED math test need to prepare for its focus on a broad mixture of mathematical content including number topics, geometry, data, and algebra. The vast majority of GED math items are presented alongside text, graphics, or charts that require students to determine based on the context what operations, if any, are needed to solve each problem. This is true even for the algebra items which often involve functions or formulas that students may need to determine or use in connection with a written, realistic situation.[9] Scientific calculators may be used on half of the GED exam and, beginning in 2012, on the entire exam.[10] The exam is paper-and-pencil, timed, and allows students to do the problems in any order they wish.

The COMPASS math exams used at CUNY are vastly different in content and context from the GED math test. According to available sample items from the publisher, the COMPASS pre-algebra section has some similarities to the GED math test in that both can involve fractions, decimals, percents, and calculations of arithmetic means. What is different about the COMPASS exams at CUNY is that the arithmetic items are presented without access to a calculator as a purer test of computation ability than would likely appear on the GED. The COMPASS exams were actually created for students who have access to approved four-function,

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[8] *2007 GED Program Statistical Report*, The GED Testing Service, pages 54-57.
[9] Author's analysis of 175 items contained in *GED Mathematics Official GED Practice Test*, The GED Testing Service, distributed by Steck-Vaughn Company; Form PA, PB, and PC (2001); Form PD and PE (2003); Form PF and PG, (2007).
[10] *The GED Mathematics Test: Comparison of 2012 and 2002 Series Frameworks* by The GED Testing Service.

scientific, and even graphing calculators, but CUNY does not permit students to use calculators of any kind.[11] While geometry and especially data and graph interpretation are significant content areas on the GED math exam, the critical two COMPASS exams do not appear to include any topics in these areas beyond arithmetic means. The COMPASS algebra section with its heavily abstract approach to rational expressions, factoring, functions, and equation-solving is very different from the more contextualized approach to algebra on the GED. The COMPASS exams are computer-adaptive, un-timed, and require students to answer one question at a time as the software adjusts to student responses.[12]

Math is a significant challenge for many students studying to take the GED exam. Even for students who are successful in that exam, though, many continue to have deep math weaknesses. Because of poor exam alignment, we should not expect that GED math preparation alone will also equip students to do well on the COMPASS exams. CUNY data has shown this to be the case by relating GED subject test scores to the likelihood that students passed the COMPASS exams by the end of their first semester of college study. GED math scores were found to only account for about 24% of the variability in students reaching math proficiency by the end of their first semester.[13]

There are some indications that the planned 2012 reforms to the GED exam may improve GED-COMPASS math alignment[14], but any change in this direction likely will be modest, and adult literacy math teachers may lack the training and math content knowledge to skillfully teach more challenging, abstract algebra topics.

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[11] COMPASS calculator-use guidelines are explained at http://www.act.org/compass/sample/math.html. Colleges and universities around the country are not uniform in their approach to calculator use on these tests. While CUNY does not allow them, the Chicago City College system permits calculator use on all COMPASS math exams. Based on my reading of the small number of items that have been released from ACT, Inc., it would appear that calculators would be the most useful on the pre-algebra section with items involving fractions, decimals, percents, arithmetic means, and square roots and not very helpful on the algebra section. It does not appear that ACT, Inc. has produced separate exam software for institutions that do not permit calculators. Colleges that do and do not allow calculators all share links to the same ACT-produced sample test questions.

[12] *COMPASS Sample Test Questions—A Guide for Students and Parents, Mathematics*, ACT, Inc., 2004.

[13] *College Readiness of New York City's GED Recipients*, CUNY Office of Institutional Research and Assessment, 2008, page 13.

[14] *The 2012 Series GED Test—Mathematics Content Standards* published by the GED Testing Service, February 2009, located at http://www.acenet.edu/Content/NavigationMenu/ged/ContentStandards2012_Math.pdf

Remedial Math Outcomes at CUNY

A reasonable person could look at the low pass rates on the COMPASS math exams for GED graduates and ask the following:

"What's the big deal if students fail one or both of the COMPASS math exams? Can't they enroll in remedial math courses for one semester, or two at the most, where they will get the help they need until their math is up to speed?"

Unfortunately, instead of efficiently building the mathematical skills and reasoning needed for more challenging courses, a large number of CUNY students who initially place into remedial math courses struggle to ever pass those courses. This difficulty in remedial math courses is also related to a decreased likelihood of remaining in college.

Pass rates on the pre-algebra and algebra placement exams for students entering CUNY were given in a previous section. Ten CUNY colleges offer remedial math courses to students who fail one or both of these exams.[15] For this discussion, we will call these courses Arithmetic and Elementary Algebra. The following chart shows the success rates for all students who *completed* Arithmetic and Elementary Algebra courses in the fall 2004 and fall 2007 semesters. "Success rates" here refer to the share of students who earned a grade of C- or higher.

Success Rates for Students Who Completed Remedial Math Courses at CUNY[16]

	Success Rate in Arithmetic	Success Rate in Elementary Algebra
2004	58%	53%
2007	47%	48%

Fewer than half of the students who completed Arithmetic or Elementary Algebra in 2007 earned a grade of C- or higher. However, significant numbers of students who enroll in math courses at CUNY withdraw before completing the semester and are not counted in these figures. Many times, this occurs when a student is discouraged, struggling, and is unlikely to pass. To get a better measure of the share of students who are successful in math courses, CUNY researchers calculate the ratio of the number of students who pass math courses to the number who *start* those courses. See pass rates below for students who started Arithmetic and Elementary Algebra courses in 2007.

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[15] Of the 10 CUNY colleges offering Associate's Degree programs, six are community colleges and four are known as "comprehensive" colleges because they also offer Bachelor's Degrees.
[16] This data was reported by the CUNY Office of Institutional Research and Assessment.

Pass Rates for Students Who Started Remedial Math Courses at CUNY, 2007[17]		
	Pass Rate in Arithmetic	Pass Rate in Elementary Algebra
All students	38%	36%
Students who were repeating the course	30%	25%
Students who passed Arithmetic before taking Elementary Algebra		32%

These statistics show just over one-third (36%) of all students who started an Elementary Algebra course in the 2007 fall semester passed it. Among students who already failed Elementary Algebra and who were repeating it, one-quarter (25%) ended up passing. Students who passed the Arithmetic remedial course (after initially failing both COMPASS math exams) were unlikely to pass Elementary Algebra (32%). It is clear from this data that for many students, failing one or two of the COMPASS math exams often means more than just one or two semesters of remedial math courses.[18]

The previous data has revealed that a large share of CUNY students spend significant amounts of time, money, and financial aid taking, failing, and repeating remedial math courses. This certainly extends the time it takes for students to earn a degree. Multiple semesters of remediation can also impact students' longer-term eligibility for financial aid. More concerning, though, are the findings in a 2006 study that suggest students who struggle in remedial math courses (both GED and high school graduates) have reduced chances of remaining in college.

CUNY researchers compared the number of students who failed math courses one semester to the number of students repeating those same courses in the subsequent semester. Elementary Algebra had the lowest ratio of "repeaters" to "failures" (38%), suggesting to the authors of the study that "failing students in Elementary Algebra tend to drop out of college at a higher rate than failing students in the other classes under consideration."[19]

One- and two-semester retention rates for freshmen students who took Elementary Algebra in the fall 2003 semester provide more evidence that success in Elementary Algebra is linked to retention in college.

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[17] *Ibid*.
[18] The previous two tables were constructed using 2007 data when the COMPASS minimum passing scores were lower than they are now. Students may pass a remedial math course only when they also pass the appropriate COMPASS math exam and so while the data is not yet available, pass rates beginning in the fall 2008 semester could be lower than those shown above.
[19] *Performance in Selected Mathematics Courses at The City University of New York: Implications for Retention* by Geoffrey Akst in collaboration with the CUNY Office of Institutional Research and Assessment, 2006, page 18.

One- and Two-Semester Retention Rates for Full-Time, First-Time Freshmen Taking Elementary Algebra in the Fall 2003 Semester[20]	
	Retention Rate
Retained in Spring 2004 (one-semester retention)	
Failed Elementary Algebra in Fall 2003	77.7%
Passed Elementary Algebra in Fall 2003	90.0%
Retained in Fall 2004 (two-semester retention)	
Failed Elementary Algebra in Fall 2003	62.3%
Passed Elementary Algebra in Fall 2003	77.4%

Other retention data specific to GED students showed that nearly 40% of GED enrollees earned no credits in their first semester, either because they failed any credit courses they took or because they only enrolled in remedial courses. Almost half of the students who earned no credits did not enroll in the subsequent semester.[21]

For some, the most significant measures of retention are graduation rates. These rates are low for students in general in Associate's Degree programs at CUNY, but are lower for GED graduates when compared to New York City high school graduates.

Rate that first-time freshmen entering CUNY at the Associate's level in the Fall 2001 semester earned any kind of degree or certificate in four years[22]	
	4-Year Graduation Rate
GED graduates	12.1%
New York City high school graduates	17.9%

Even though many GED graduates have substantial math weaknesses and these weaknesses appear to play a role in retention, I am not suggesting that math is the only significant challenge facing GED graduates at CUNY. Some students may do well in math but struggle to reach reading or writing proficiency, or they may arrive

[20] *Ibid*, page 65.
[21] *College Readiness of New York City's GED Recipients*, CUNY Office of Institutional Research and Assessment, 2008, page 3.
[22] *Ibid*, page 18. These percentages rise when graduation rates are measured after more years of study, but even in these cases the rates are low. According to the CUNY Office of Institutional Research and Assessment, for all full-time, first-time freshmen entering Associate's Degree programs in 1998, 18.8% earned a Bachelor's or Associate's Degree after four years, 27.4% did so after six years, and 32.4% did so after ten years. For freshmen entering in fall semesters 1999 through 2002, four-year graduation rates (Associate's or Bachelor's) were all between 17% and 19% and six-year graduation rates were all between 26% and 29%.

unprepared for the intensity and complexity of college coursework. Factors beyond the campus such as work, family, and financial obligations also can make college continuation a great challenge, and these complicating factors have been shown to be more prevalent among GED graduates than for CUNY students in general.[23]

What is to be done?

As the Math Staff Developer for the CUNY Adult Literacy/GED Program, I work as a member of a team of staff developers who support staff and curriculum development projects for basic education, GED, and ESL classes at 14 CUNY campus Adult Learning Centers. A few years ago, this team became increasingly concerned about the large numbers of GED graduates who were struggling to complete CUNY college degree programs. We could have looked at the issues and concluded that our job (and funding base) was limited to helping students earn their GEDs. Instead, we decided to do more to help students not only earn the credential needed to enter college but to also be successful there. One of our strengths in approaching this work is that we know a great deal about adult students' academic strengths and weaknesses as well as a range of pedagogical methods that are effective with the GED student population. In addition, as a part of The City University of New York, we can develop relationships with faculty to better understand the demands of college work. It was only logical that we would try to do something to facilitate a more successful transition to CUNY for New York City GED graduates.

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[23] Survey results in the report *"College Readiness of New York City's GED Recipients"*, prepared by the CUNY Office of Institutional Research and Assessment in 2008 revealed that GED graduates were more likely than other students to work 20 or more hours per week and were twice as likely to provide care to others 20 or more hours per week. GED graduates were also more likely to report wanting their college to offer more night classes.

A Brief History of the CUNY College Transition Program[24]

Beginnings

Photo by Sam Seifnourian

Jackie and Roxanne discuss a CTP math problem.

The CUNY Adult Literacy/GED Program offered its first College Transition Program (CTP) math class in the spring 2007 semester. Students were recommended for the class by teachers in CUNY campus Adult Learning Centers. The students committed to attend class one day per week to focus on pre-algebra and algebra content related to the COMPASS exams. A few of the students already had their GED when the course began, but most were attending GED classes outside of CTP for an additional three or four days per week. I was the principal instructor for the 13-session, 39-hour math course. Also in the spring 2007 semester, fellow staff developers Gayle Cooper-Shpirt and Hilary Sideris began teaching a CTP reading/writing class. That class included a few students from the math class, but most were students we did not share.

Four Semesters

Over four semesters from spring 2007 through summer 2008, CTP mounted a total of seven math classes and five reading/writing classes. Our experiences in those semesters led us to a set of conclusions about how CTP needed to be remade in virtually all areas.

- Significantly greater instructional hours were needed in both content courses. This was a view shared by the instructors and most students.
- Students had the choice of taking one or both academic classes based on their self-assessment of need, but CTP staff developers felt that virtually all of the students would benefit from taking both content classes, even when a student had strengths in one area.
- In some instances, students would complete a semester of CTP but still require another semester or year to pass the GED exam. This would cause an unfortunate time gap between students' CTP class(es) and their CUNY placement exams. We did not want CTP classes to compete with the time students needed to focus on the more immediate goal of a GED, especially when we were contemplating a significant increase in CTP instructional intensity. We concluded students should hold a GED before joining the program.
- Many students had difficulty navigating the admissions, financial aid, and other enrollment challenges at CUNY on their own. Students were hungry for guidance on how to complete these tasks as well as to make decisions on selecting a college, a major, and classes. We needed a comprehensive approach to advisement to assist CTP students in application and enrollment processes, advocate for them when necessary, and help them make informed decisions about their educational future.[25]

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[24] The CUNY College Transition Program (CTP) restructured and beginning in the fall 2009 semester became known as the College Transition Initiative (CTI).

[25] A strong advisement component is widely seen as a critical practice in developmental education programs. Boylan and Saxon in the 2002 article "*What Works in Remediation—Lessons From 30 Years of Research*" for the National Center for Developmental Education point to several studies that found successful remedial education programs had a "strong" counseling component, and they noted this counseling was most successful when it was integrated in the overall program,

A New Model Is Unveiled

In the fall 2008 semester, we chose to simultaneously test a whole set of academic and advisement innovations in the CTP class at the LaGuardia Community College Adult Learning Center.

- We changed the CTP admissions standard. Students would now need to hold their GED at the start of the CTP semester. We did not screen students based on their GED scores, but we did maintain our earlier practice of trying to select students who were recommended with a record of decent attendance and work habits in a GED preparation program. [*See Appendix A for GED score data that compares CTP students to GED graduates in general who enroll in CUNY.*]
- We significantly increased the number of instructional hours. Each content course (math and reading/writing) would meet six hours per week. For the semester as a whole, there were 72 math instructional hours and 72 reading/writing instructional hours.
- We instituted a learning community model in which the same students were scheduled for both content courses and for group advisement sessions. All students would attend the program four days per week—two days for math, one day for writing, and one day for reading (which also included writing in response to texts). The reading and writing components had some shared practices and curricular goals, but no attempt was made to link the reading or writing curricula to the mathematics curriculum.
- We transformed academic advisement. An academic advisor organized all-group sessions to assist students in doing their on-line college applications, financial aid applications, and to do cohort-based CUNY placement testing. The advisor also led weekly, hour-long meetings to educate students about credits, tuition, the GPA, enrollment requirements (such as proof of immunizations and residency), how to choose a college and a course of study, time management, and more. The advisement model was highly proactive with frequent "check-ins" with individual students to be sure they were completing necessary enrollment tasks.

Outcomes

The LaGuardia CTP students were retained, applied to CUNY, and completed placement testing in large numbers. [*See Appendix B for math retention, application, and testing data for the combined fall 2008 and spring 2009 CTP cohorts.*]

LaGuardia students made strong gains in internal math assessments over the semester, indicating that students improved in their ability to do the math that we were teaching them. To allow for comparison, math pre- and post-tests were carefully designed to include the same skills, reasoning, and difficulty for parallel items. All CTP math assessments are constructed to measure student understanding of the topics studied in CTP and do not attempt to assess all possible COMPASS math topics.[26] Test averages are shown below for the LaGuardia students. [*See Appendix C for math assessment data for the combined fall 2008 and spring 2009 CTP cohorts.*]

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was undertaken early in the semester, and was carried out by trained staff, among others elements. Also see "*Toward a More Comprehensive Conception of College Readiness"* by David Conley and published by the Bill and Melinda Gates Foundation, 2007, page 17, for a discussion of the importance of developing students' "college knowledge".

[26] See the section titled *Math Content in the College Transition Program* for the content choices we made in the course.

Internal Math Assessments for the Fall 2008 LaGuardia CTP Class	
Pre-Test average (30 testers)	28.4%
Post-Test average (27 testers)	77.8%

Other math outcomes for CTP students that are more difficult to measure are worth noting here. Several of the students who began with deep math weaknesses and insecurities gained not only in their number and algebra abilities, but also in their belief that they could learn math, do math, and participate in mathematical conversations. We believe that our approach to teaching and learning contributes to these changes in "productive disposition", and while they are more difficult to measure than test scores, we should consider ways of capturing these changes using qualitative techniques.[27] Ultimately, we believe these changes will lead to quantifiable results—namely, greater persistence and success in college math courses (remedial or credit-based) for CTP students when compared to typical GED graduates who move directly into CUNY.

Student cooperation grew immensely over the course. As students built friendships within the learning community, and probably also because the reading, writing, and math instruction encouraged frequent collaboration, LaGuardia CTP students increasingly supported one another in academic and non-academic ways inside and outside of class. This cooperation has continued for many of the students in their first college semester. Research has shown that students' willingness to collaborate with other students outside of class can be critical for success in college mathematics, especially for students of color.[28]

The combined effect of the new academic and advisement models appeared to have strong effects on course retention, academic placements in the subsequent semester, rates of college admission and financial aid completion, academic improvement in the course, and in building a culture of support among the students. Of course, we were intensely interested to see how our students would perform on the CUNY placement exams at the end of the semester. These results have been encouraging.

Based on the strong early results from the first intensive CTP cohort, we extended the intensive model in the spring 2009 semester to include classes at LaGuardia Community College and Borough of Manhattan Community College (BMCC). Combined initial placement test results for three CTP intensive classes are shown below and are compared to typical GED and non-exempt high school graduates who test at CUNY.
[*See Appendix D for detailed placement test results for students in all three cohorts.]*

[27] "Productive disposition" was described in the book Adding It Up: Helping Children Learn Mathematics, by Jeremy Kilpatrick, Jane Swafford, Bradford Findell as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics." The authors include productive disposition among five intertwined strands of math proficiency that also include procedural fluency, conceptual understanding, adaptive reasoning, and strategic competence. According to the authors, students' productive disposition "develops when the other strands do and helps each of them develop." More will be said about these strands in a later section of this paper. Adding It Up was published in 2001 by the National Research Council.

[28] *Studying Students Studying Calculus: A Look at the Lives of Minority Mathematics Students in College* by Uri Treisman in *The College Mathematics Journal*, Volume 23, #5, 1992, pages 362-372.

Pass Rates on Initial CUNY Basic Skills Exams for GED Graduates, Non-Exempt NYC High School Graduates, and CTP Students Entering Associate's Programs at CUNY[29]			
	GED Graduates Fall 2008	Non-Exempt NYC Public H.S. Graduates Fall 2008	CTP Students Fall 2008 and Spring 2009
Pass rate in reading	70.1%	47.0%	76.0%
Pass rate in writing	25.9%	21.1%	72.0%
Pass rate in math part one (pre-algebra)	59.3%	41.3%	87.5%
Pass rate in math part two (algebra)	14.0%	14.3%	51.0%
Pass in all areas	3.8%	1.5%	32.7%

Almost one-third of CTP students passed all the placement exams. This is a striking statistic not only because it is so rare for typical GED graduates entering CUNY, but also because recent data has shown that it typically takes one year of college study before 33% of a cohort of GED graduates reaches proficiency in all areas.[30]

Mean COMPASS algebra scores may reveal additional strengths in CTP student results that are not visible in pass/fail rates. The chart below shows that CTP students who failed the algebra exam tended to do so with higher scores than typical GED and high school graduates. It suggests that CTP students who still need to take the remedial algebra course may be better prepared than other students to pass that course.

Mean Scores on COMPASS Algebra for GED Graduates, NYC High School Graduates, and CTP Students Entering Associate's Programs at CUNY[31]			
	GED Graduates Fall 2008	NYC H.S. Graduates Fall 2008	CTP Students Fall 2008 and Spring 2009
Math part two average score (passing score = 30)	23.0	22.6	37.3
Average score for those who failed math part two (passing score = 30)	19.7	19.9	24.0

[29] The data on GED and high school graduates is from Collaborative Programs Research and Evaluation and refers to non-exempt testers who applied through the central application system and not those who applied through direct admission. For GED graduates, $n = 1{,}230$ and for high school graduates, $n = 6{,}469$. For mathematics, we use the official CUNY minimum passing scores for both COMPASS exams (30 on parts one and two). A few students are not counted in these rates if they had exemptions in one or more exams or because they entered certificate rather than Associate's Degree programs. For each of the pass rates, at least 48 and at most 50 student scores were available for the calculations.

[30] "*College Readiness of New York City's GED Recipients*", CUNY Office of Institutional Research and Assessment, 2008, page 22. The 33% statistic applies to GED graduates at CUNY in the 2001-2002, 2004-2005, and 2006-2007 cohorts.

[31] The data on GED and high school graduates is from Collaborative Programs Research and Evaluation and refers to non-exempt testers who applied through the central application system and not those who applied through direct admission. For GED graduates, $n = 1{,}230$ and for high school graduates, $n = 6{,}469$. For mathematics, we use the official CUNY minimum passing score for the COMPASS algebra exam (30). For CTP student figures, $n = 51$ for the overall average and $n = 26$ for the average among those who failed part two.

The early results are encouraging, but we should be careful not to draw too many conclusions from what is still a small number of students. This is also not a randomized sample of GED graduates. Even though we did not take students' particular GED scores into account in admitting students to CTP (and the GED score profile of the cohorts reflect this), we did try to include students who had a habit of good attendance in their GED preparation program.

We will continue to follow these and subsequent CTP student cohorts through their college study at CUNY, reporting data on students' GPAs and rates of credit accumulation, retention, and graduation.

Content of the COMPASS Math Exams

As was shown in an earlier section, results from COMPASS math exams have a significant impact on the time it takes and even the likelihood that a student will earn a degree at CUNY. In planning a transition math course for GED graduates, it was important for us to learn what we could about the math content of the COMPASS exams. Unfortunately, very little information is available compared with what is available to instructors preparing students for the GED or New York State Regents math exams.[32]

The only widely-available information on the content of the COMPASS math exams released by ACT, Inc., the publisher of the exams, is a document that includes a list of "content areas" and 30 practice items. This document states that a "majority" of test items are drawn from the following:

Content Areas for the COMPASS Math Exams, parts 1 and 2[33]	
Part 1: Pre-Algebra Content Areas	*Part 2: Algebra Content Areas*
Operations with integers	Substituting values into algebraic expressions
Operations with decimals	Setting up equations for given situations
Operations with fractions	Basic operations with polynomials
Positive integer exponents, sq. roots, and sci. notation	Factoring polynomials
Ratios and proportions	Linear equations in one variable
Percentages	Rational expressions
Averages	Linear equations in two variables

On their own, lists of topics like these are not very useful in getting to know an exam or helping students prepare for it. A skilled math instructor could create problems within every one of these areas that are radically different in format, context, and complexity. Sample items are a critical additional way of gaining insight into the math content valued by an exam publisher. Remembering that the COMPASS math exams are computer-adaptive, a student's scaled exam score is determined using a combination of the number of correct items and some measure of their difficulty. Unfortunately, the 30 official COMPASS practice items have no accompanying rubric that would help us understand how many and which problems would need to be answered correctly in order to earn a passing score. It is more confounding when we see that the sample items have wildly different levels of difficulty. Simply, we do not have good information on the content of the COMPASS math exams, and especially the level of math content knowledge that is needed to earn a passing score.

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[32] The GED Testing Service has published seven half-length math practice tests that include a total of 175 items. The New York State Education Department releases complete Regents math exams every semester on its website after they have been administered.

[33] *COMPASS Sample Test Questions—A Guide for Students and Parents, Mathematics*, ACT, Inc., 2004, pages 1-10. The ACT/COMPASS website also includes an additional eight practice items at http://www.act.org/compass/sample/math.html.

Anyone can find "COMPASS math practice" materials on the internet—links to these materials are even housed as a part of CUNY college websites. We should remember that except for the 30 practice items already mentioned, all other items have been created by observers' best guesses about COMPASS math content. Students or instructors might visit these sites and believe the problems represent the content valued by the creators of the COMPASS exams, but we cannot be certain about this.

Not only do we have very little good information on what students need to know in order to pass the COMPASS math exams, the test results do not provide much useful information on student performance. CTP students have reported that their exams were ended by the software after as few as ten questions. It is hard to imagine that the responses from ten multiple-choice items can tell us much about a student's math ability. The feedback on a student's exam is a scaled score between 0 and 100 for each exam part. No item or other analysis is provided to instructors or to the student. ACT, Inc. has produced seven diagnostic tests for the pre-algebra exam and eight diagnostic tests for the algebra exam which are available to CUNY college math departments, but these tools do not appear to be in wide use. Some have argued that CUNY should move away from the COMPASS to another software product that can simultaneously give broad placement information along with more detailed item analyses of students' precise math weaknesses. In working with any diagnostic exam, we should remember that utilizing the results to modify instruction for a classroom of students can be a challenging task, especially when students' individual areas of weakness do not neatly coincide.

Teaching and Learning in Remedial Math Classes

Before describing CTP math teaching and learning practices, it can be helpful to review more traditional teaching practices. I am not aware of any study that has attempted to sample and describe typical instruction in remedial math classrooms inside or outside of CUNY. Despite this, I will detail practices that appear to be common in remedial math courses (especially remedial algebra) that are based on references to typical instruction in reports and research, my conversations with math faculty, department chairs, students, administrators, and researchers, and from my review of selected remedial algebra syllabi.

Remedial algebra curricula typically include coverage of a vast number of topics. The most striking and consequential feature of remedial algebra courses can be the large quantity of topics covered. The instructional pace needed to teach so many topics limits how material may be presented and how much student communication about the ideas can occur in the classroom. A fast pace makes it challenging for the instructor and students to explore topics deeply, including ones that have great potential richness or that are particularly difficult for students. When math or basic skills departments require remedial instructors to follow departmental syllabi and administer common exams, the instructors may not feel they have the flexibility to slow down and consider topics more carefully when students need it. Lloyd Bond has argued, and I certainly agree, that common exams provide important opportunities for curriculum and faculty development, but common syllabi and exams can also pressure instructors to conform to a coverage-first approach. [34]

Mathematical ideas are often presented through lecture. With little time and many topics to cover, lecture can appear to be the most efficient method of presenting mathematical ideas. In this approach it can be the instructor who is really doing the math while the students are more passive note-takers. When the majority of class time is devoted to instructor presentations, there is less time for students to do problems, raise questions, make and explore errors, show confusion, or consider multiple ways of looking at mathematical ideas.

Memorization of rules and procedures is emphasized. Emphasizing math rules can be seductive to an instructor because it does not take long to express them--*"In this case you add the exponents."* Because students may not acquire a deep understanding of the mathematics that underlies these rules, their understanding is often fragile and the rules can be forgotten or misused.

Remedial math instructors are given an enormously challenging task. Many students enter remedial classrooms with profound math weaknesses, but the pacing and type of instruction may be more appropriate for students who only need a “brush-up”. I believe the practice of moving rapidly through many math topics, and the limits this puts on pedagogy, can be viewed more broadly as a continuation of a common approach to school math instruction in the U.S.[35] Teachers in many middle and high schools feel similar pressure to move quickly to prepare students for that year’s standardized tests. Students in those settings may not develop strong math

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[34] “*The Case for Common Examinations*” by Lloyd Bond was printed in *Perspectives* on the Carnegie Foundation for the Advancement of Teaching website: www.carnegiefoundation.org. In “*Technology Solutions for Developmental Math—An Overview of Current and Emerging Practices*”, a 2009 report for the William and Flora Hewlett and the Bill and Melinda Gates Foundations, Rhonda Epper and Elaine Baker describe how many topics and limited instructional time may prevent faculty from being able to develop both students’ procedural abilities and conceptual understanding. Pasadena City College Project Director Brock Klein is quoted in the article saying “the content/coverage issue is single most common reason math instructors give for not transforming their practice…They claim they do not have time to be innovative. They have to cover ten chapters.”

[35] In *“A Coherent Curriculum: The Case of Mathematics”* published in the Summer 2002 issue of *American Educator*, William Schmidt, Richard Houang, and Leland Cogan draw on the Third International Math and Science Study (TIMSS) to argue that school math teachers “work in a context that demands that they teach a lot of things, but nothing in-depth. We truly have standards, and thus enacted curricula, that are a ‘mile wide and an inch deep’…the teachers in our country are simply doing what we have asked them to do: ‘Teach everything you can. Don’t worry about depth. Your goal is to teach 35 things briefly, not 10 things well.’”

understanding and often wind up needing to study the same topics again the following year. By the time students enter college, they have seen many of these remedial math topics several times in prior years without managing to master them. Low success rates in college remedial math courses may signal a continuation of that unfortunate history.

Curriculum and staff development can be limited. Remedial math instructors generally receive a syllabus and textbook to guide their work. Instructors do not typically have continuing, structured, and supported opportunities to come together to observe, analyze, and discuss methods for teaching individual topics outlined in the syllabus. Curriculum and staff development projects that include significant numbers of adjunct faculty are even more rare. While there are pockets of faculty collaboration over curriculum and pedagogy around the country, these innovations appear to affect a small share of total remedial math instructors and students.[36]

The role of certain technologies is increasing. In recent years, many colleges are turning to or are broadening their use of computer software as a supplement or even a replacement for live remedial math instructors. A review of the research on the learning effects of this technology has shown mixed results.[37] In contrast to the attention that computer software has gained, graphing calculators are rarely mentioned in recent reports on the use of technology in remedial math instruction, and they are unlikely to be found in the vast majority of remedial algebra classrooms. The near silence on graphing calculators exists despite the 1995 and 2006 teaching and learning standards devised by The American Mathematical Association of Two-Year Colleges (AMATYC) which have asserted that graphing calculators can be powerful learning tools, and that developmental math students should have experiences with them alongside other technologies.[38]

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[36] An example of innovation at CUNY is Project Quantum Leap at LaGuardia Community College where math and other faculty are engaged in a multi-year effort to infuse authentic scientific concepts into remedial math lesson-planning.
[37] "*Strengthening Mathematics Skills at the Postsecondary Level: Literature Review and Analysis*", pages 27-33, prepared for the U.S. Department of Education Office of Vocational and Adult Education, Division of Adult Education and Literacy, 2005.
[38] "*Crossroads in Mathematics: Standards for Introductory Mathematics Before Calculus*", AMATYC, 1995, page 11 and "*Beyond Crossroads: Implementing Math Standards in the First Two Years of College*", AMATYC, 2006, page 42.

Math Content in the College Transition Program

In selecting math content for CTP, we have needed to consider who our students are and what math experiences and habits they bring with them. A significant number of GED graduates have deep math weaknesses, and this usually coincides with a fear and dislike of math and math learning. GED graduates' math ability is not uniform, however, and so CTP classes have typically included a mixture of students who would likely fail both COMPASS exams without our intervention, some who are primarily weak in algebra, and some who have strengths in both areas. CTP math classes also include significant numbers of students whose first language is not English and who must contend not only with the mathematics but also with the vocabulary, notation, and other conventions of English-language math classrooms.

As a part of selecting and refining the math content for the CTP math course, we have gathered and considered the following information:

- Data on GED graduates' typical performance on individual COMPASS math exams
- Available information on the content of the COMPASS math exams from ACT, Inc.
- College remedial math syllabi
- Our sense of the content that can be reasonably studied in a CTP semester given the depth of understanding we wish to achieve
- The mixture of learners, language backgrounds, and math histories of our students
- Student and instructor reflections each semester
- Data on students' performance on CTP internal assessments
- Data on students' performance on the COMPASS math exams
- Data on former CTP students' performance in college math classes

Taking these factors into account, we made the early decision that CTP math content needed to include a mixture of number topics, functions topics, and what we call "elementary algebra" topics.[39] Even though the majority of GED graduates pass the COMPASS pre-algebra exam, many of them have deep number weaknesses and need further instruction in this area. In some cases, these number weaknesses are at the root of their algebraic weaknesses, and in other cases, we want to emphasize number relationships to illuminate more abstract work with variables.[40]

Rather than organizing the content in a more traditional way with all number topics first and all algebra topics later, we have integrated number, elementary algebra, and functions topics throughout. Mixing the content helps students to make important connections between topics, and the variability contributes to a more vibrant classroom for the instructor and students. As soon as a number topic is considered, we may incorporate it into our work with expressions and functions to increase the challenge, or we may use it as a basis for introducing a new algebraic idea.

One of the most important decisions we made was to break from an instructional model that emphasizes covering a vast number of topics because it has not proven successful for many students. In choosing to emphasize students' depth of understanding and the ability to think and communicate like scientists, we have accepted that we cannot study several topics that are normally included in remedial math courses at CUNY. Even when we include a topic that is found on a remedial algebra syllabus, we may consider a narrower set of

[39] "Elementary algebra" here refers to work with expressions, polynomials, and equations that are not necessarily related to functions.
[40] An example of a number weakness that affects students' algebraic skills and reasoning is integer arithmetic. Many GED graduates have not mastered integer arithmetic and this is an area that must be strengthened if they will be successful working with functions, simplifying or factoring expressions, and solving equations, among other common algebra tasks.

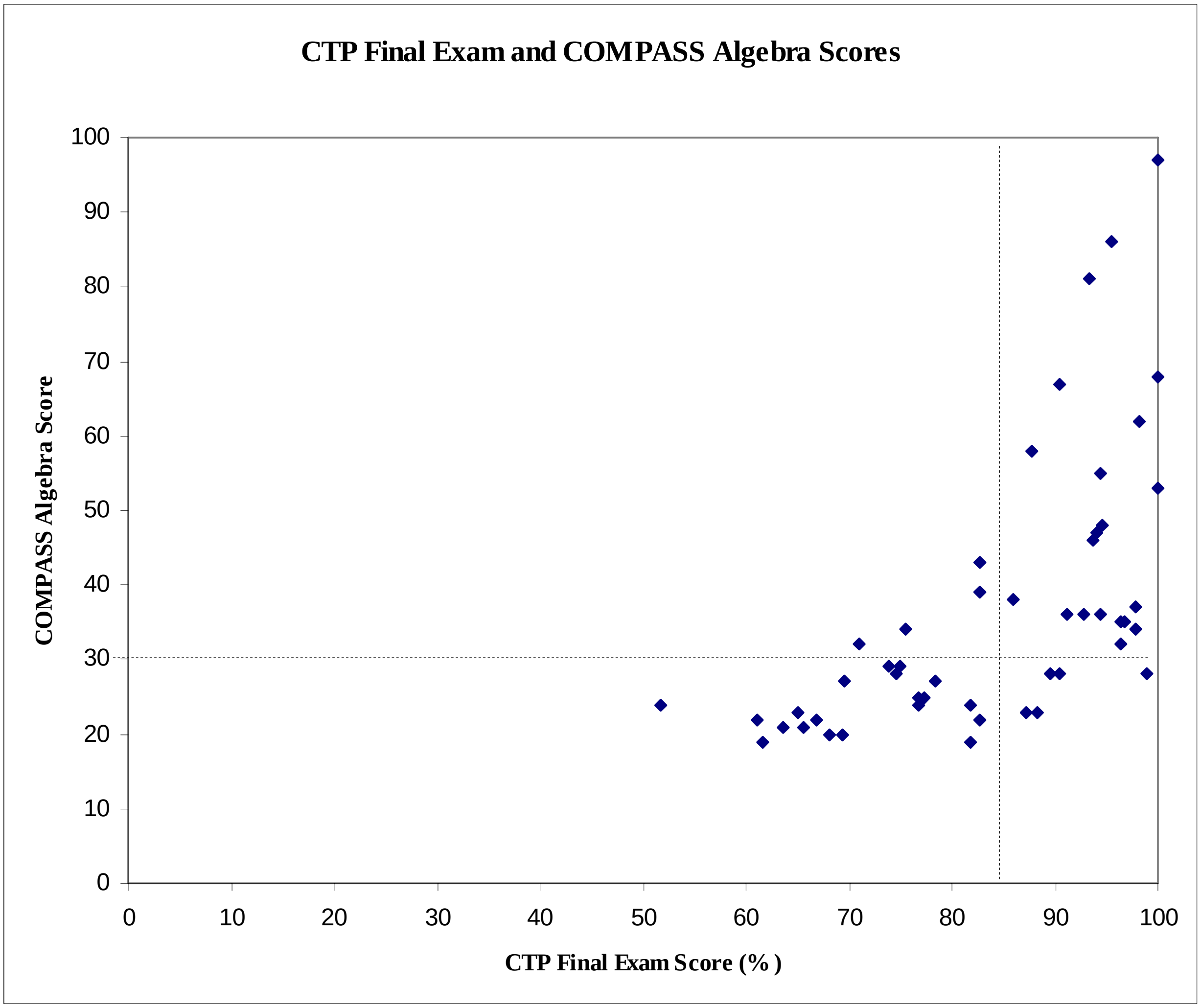
concepts within that topic.[41] We do not see this narrowing of topics as a "dumbing down" of the curriculum. On the contrary, we see the careful development of a smaller number of topics as a way of taking students very seriously as math learners. Despite the significant pressures that exist around various high-stakes tests, some notable middle, high school, and college math programs are also resisting a coverage-first approach.[42]

In some instances, we include activities and content we know have no direct relation to the COMPASS exams. An example is our work with functions on the TI-83+ graphing calculator. Graphing calculators are ubiquitous in high school math classrooms and are used in pre-calculus, college algebra, and some statistics courses at CUNY. It is our feeling that adult students deserve some experience working with this technology. Our adult students also enjoy working with the calculators and they give us opportunities to explore complex, realistic functions. Graphing calculators, like all calculators, are not permitted on the COMPASS math exams at CUNY, but preparing for the exams is not our only goal.

We have compared student performance on CTP internal assessments and the COMPASS algebra exam to help us understand what level of CTP math ability may be correlated with COMPASS algebra success. This is important because a strong connection between the assessments could indicate that we are helping students to learn the content that is also valued by ACT, Inc. The following graph shows CTP final exam scores measured against COMPASS algebra scores for all students from the three intensive cohorts for whom we have complete data.

[41] As an example, CTP students study systems of linear equations. We introduce the idea using two functions in a realistic context and students discuss the similarities and differences between ordered pairs, function solutions, and system solutions by making references to the realistic scenario. Students then learn to identify system solutions from lists of ordered pairs, in tables of values, and by graphing. Still, we do not teach two common techniques for determining system solutions, commonly known as “elimination” and “substitution”.

[42] Susan Goldberger in the report “*Beating the Odds: The Real Challenges Behind the Math Achievement Gap—And What High-Achieving Schools Can Teach Us About How to Close It*”, written in 2008 for the Carnegie-IAS Commission on Mathematics and Science Education, describes how the math faculty of the College Park Campus School (CPCS) in Worcester, Massachusetts made the decision to attach primary importance to improving students’ conceptual understanding rather than coverage of topics in their math curricula. CPCS is ranked among the best schools in the state despite accepting large numbers of underprepared math students in the 7th grade. In the June 8, 2009 edition of *The New York Times*, the article *“Connecticut District Tosses Algebra Textbooks and Goes Online”* described how the math faculty at the high-performing Staples High School in Westport, Connecticut was given permission to cut the number of topics in the two-year algebra curriculum in half to improve student understanding and to limit the need for re-teaching. Rhonda Eper and Elaine Baker in “*Technology Solutions for Developmental Math—An Overview of Current and Emerging Practices*” describe an instance at Pasadena City College where the number of pre-algebra concepts was reduced by one-third so that practical applications could be provided for essential concepts. In this case, retention and success rates increased. Interestingly, the students receiving the narrower, deeper approach fared as well in the following math course as those who were taught more topics in the traditional manner.



Fifty (50) students from three intensive CTP classes had CTP final exam and COMPASS algebra scores for use in this plot. The COMPASS passing score of 30 is shown using a horizontal line. The vertical line shows a score of 85% on the CTP final exam. This score appears to be a useful predictor for COMPASS algebra success. See a summary of the data in the charts below.

Number of students who scored **85% or higher** on the CTP final exam	Number of these who passed the COMPASS algebra exam	Percent who passed the COMPASS algebra exam
26	21	80.8%

Number of students who scored **less than 85%** on the CTP final exam	Number of these who passed the COMPASS algebra exam	Percent who passed the COMPASS algebra exam
24	4	16.7%

This data suggests that the mix of math content we examine in CTP is related to the content students are facing on the COMPASS algebra exam. If students were doing very well on the CTP final exam but were routinely failing the COMPASS exam, we would be forced to question whether we were teaching a relevant mix of content or if the instructional intensity was adequate.

Using CTP assessment data, we can also show that the students who eventually passed the COMPASS algebra exam did not enter CTP already able to do the content of our course. For the 25 students who eventually passed COMPASS algebra, their mean CTP pre-test score was 46.15% and their mean post-test score was 91.97%. [*See Appendix E for a table of this score information.*]

Math Teaching and Learning in the College Transition Program

Photo by Sam Seifnourian

Tenzin at work in a CTP class.

The CTP approach to math teaching and learning has been guided by the many goals we have for our students. One goal is to reduce or eliminate students' need for math remediation. While important, this is not the only goal. CTP math is also meant to deepen students' understanding of number and algebra topics so that their learning can be extended to other and more complex content leading to success in their first college math course (remedial or for-credit). For this to happen for students who do not have a history of success in math classes, the course must increase students' confidence and persistence as math learners. Another goal is to give students regular opportunities to talk about math, be curious, and think critically so that they begin to learn and communicate like scientists. Finally, we wish to prepare students for college-level academic expectations while preserving the nurturing characteristics of an adult literacy program.

Math teaching and learning in CTP looks very different from lecture-based classrooms that feature quick coverage of topics and that focus on student recall of rules and procedures. Our approach to pedagogy has much more in common with teaching and learning practices highlighted in two National Research Council documents—How Students Learn: History, Mathematics, and Science in the Classroom[43], and Adding It Up: Helping Children Learn Mathematics.[44]

The authors of How Students Learn advocate for math classrooms:

> *"…that at the same time (are) learner-centered, knowledge-centered, assessment-centered, and community-centered…The instruction described is learner-centered in that it draws out and builds on student thinking. It is also knowledge-centered in that it focuses simultaneously on the conceptual understanding and the procedural knowledge of a topic, which students must master to be proficient, and the learning paths that can lead from existing to more advanced understanding. It is assessment-centered in that there are frequent opportunities for students to reveal their thinking on a topic so the teacher can shape instruction in response to their learning, and students can be made aware of their own progress. And it is community-centered in that the norms of the classroom community value student ideas, encourage productive interchange, and promote collaborative learning."*[45]

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[43] How Students Learn: History, Mathematics, and Science in the Classroom by the National Research Council, the Center for Studies on Behavior and Development, and the Committee on How People Learn, 2005.
[44] Adding It Up: Helping Children Learn Mathematics by Jeremy Kilpatrick, Jane Swafford, Bradford Findell, the Mathematics Learning Study Committee, and the National Research Council, 2001.
[45] How Students Learn: History, Mathematics, and Science in the Classroom by the National Research Council, the Center for Studies on Behavior and Development, and the Committee on How People Learn, 2005. *Chapter 5, Mathematical Understanding: An Introduction*, by Karen Fuson, Mindy Kalchman, and John Bransford, page 242.

Adding It Up authors have created a broad view of what it means for a student to be math proficient and go beyond discussions that focus mainly on "procedures" and "concepts". In their work, the following five strands are "interwoven and interdependent" in the process of developing proficient math students.[46]

> *Conceptual understanding*—comprehension of mathematical concepts, operations, and relations
>
> *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
>
> *Strategic competence*—ability to formulate, represent, and solve mathematical problems
>
> *Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
>
> *Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Keeping these classroom practices and elements of math proficiency in mind, one will see many connections to the pedagogical practices we have adopted in CTP math and that are described below.

Math learning that is meaningful and not "rote-ful"

The following are examples of the ways we involve students actively in their learning, move students from number and contextualized problems towards more abstract reasoning, and in general try to foster depth of understanding and confidence among CTP math students.

Rules can be the pedagogical endpoint, not the starting point. Instead of relying on an instructor to demonstrate math rules or procedures that students are expected to follow, CTP students gain confidence in new ideas by examining and discussing the underlying mathematical relationships from the beginning. After students work with an idea and develop some fluency, rules emerge based on students' own work. In this way, the rules come more often at the end of a lesson than in the beginning. It also means students who forget a rule may not be helpless—they can think about the mathematical relationships and may be able to work their way back to a solution. For students who successfully memorized some of the math rules in an earlier class, this adds important justification and depth to their understanding. In these ways we seek to build students' conceptual abilities in addition to strengthening their procedural fluency. [*See Appendix F for an example from the curriculum.*]

Focusing on mathematical relationships rather than rules is a change for many students. We have found that most of our adult students adapt well to this approach. It may help that we do not forbid students from using rules, but gently insist that students demonstrate and articulate why the rules work if they wish to use them.

Lecture is almost non-existent. Many math concepts can be introduced through a series of well-crafted questions or by calling on students' inductive reasoning to guide them from previous understandings to new ideas. The instructor plays a critical role in orchestrating these exchanges and in explicitly naming conventions that are not likely to be discovered by students. Using this approach, CTP students are not simply note-takers but are actively doing mathematical reasoning and strengthening their math vocabulary almost every step of the way to new ideas.[47] [*See Appendix G for an example from the curriculum.*]

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[46] Adding It Up: Helping Children Learn Mathematics by Jeremy Kilpatrick, Jane Swafford, Bradford Findell, the Mathematics Learning Study Committee, and the National Research Council, 2001, page 116.

[47] Numerous standards documents and reports have pointed to the importance of active, student-centered instruction. These include the *Standards for Pedagogy* outlined in "*Crossroads: Standards for Introductory Mathematics Before Calculus*", by The American Association of Two-Year Colleges (AMATYC), 1995, "*Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College*", AMATYC, 2006, the *Teaching and Learning Principles* of the Adult Numeracy Network, a national organization affiliated with the National Council of Teachers of Mathematics, and How

Functions presented in context can illuminate abstract ideas and notation. Even though functions are not likely to appear in realistic contexts on the COMPASS exams, the CTP math curriculum utilizes contextualized functions as an engaging way to move students from more comfortable number terrain to abstract work with expressions, functions, data tables, graphs, function notation, and systems of equations. This approach coincides with one of three core teaching principles highlighted in How Students Learn—namely, the importance of building new knowledge on the foundation of students' existing knowledge and understanding.[48] [*See Appendix H for an example from the curriculum.*]

Number relationships can illuminate algebraic relationships. Where possible, the CTP curriculum aims to tap into and boost students' number abilities to serve other ends. One example is our approach to studying the distributive property. Rather than introducing this concept in the traditional way with a dry, abstract demonstration of the property, we begin by asking students to do an everyday mental math calculation in which students employ the distributive property without realizing it. CTP instructors then guide students to formalize their mental math, observe related examples, and conjecture about the mathematical relationships. Naming the mathematical idea is the very last step in the process. What is particularly nice about this approach is that it begins with students demonstrating the distributive property, not the instructor. [*See Appendix I for an exposition of this example from the curriculum.*]

Students can be guided to think and learn like scientists. In facilitating discussions and calling on students' inductive reasoning, CTP instructors frequently ask students to respond to the kinds of questions scientists ask themselves all the time--*"What's going on here? Does this make sense? Is this always true, or is it a coincidence?"* We are trying to help our students adopt the intellectual habits of scientists (as well as engaged citizens)—inquisitiveness, critical thinking, looking to connect new information to previously-studied ideas, and a consistent desire for deep understanding. [*See Appendix J for an example from the curriculum.*]

Student talk that is more important than teacher talk

CTP math instructors agree that it is essential that students not only learn mathematics but also learn to communicate mathematically. Instructors use questions as one way to promote this communication, and our questioning style has been described as "relentless". The most useful questions are the ones that require explanations of student work and thinking. The question, *"What is the answer to problem #5?"* does not reveal much about student thinking unless it is followed by the question, *"What did you do to arrive at that answer?"* Consider this list of frequently-asked questions in the CTP math classroom:

- *What did you do? Why did you do that?*
- *Do you agree with what she just said? Why?*
- *Did any of you do it differently? How?*
- *What do you see?*
- *Does this remind you of anything?*

Photo by Sam Seifnourian

Araceli and Toribio talk about math.

Students Learn: History, Mathematics, and Science in the Classroom, by the National Research Council, the Center for Studies on Behavior and Development, and the Committee on How People Learn, 2005.

[48] *Chapter 8, Teaching and Learning Functions* by Mindy Kalchman and Kenneth Koedinger from How Students Learn: History, Mathematics, and Science in the Classroom, by the National Research Council, the Center for Studies on Behavior and Development, and the Committee on How People Learn, 2005, page 351-353.

CTP instructors use questions in several ways—as a substitute for lecture so that students can be guided to observe, discover, and incorporate new ideas, as a means for continuously assessing student understanding, and as a learning tool in itself because a student who is explaining an idea is deepening their understanding.

Students are expected to evaluate each other's ideas in the classroom. In this way, authority is shared between instructor and students. The physical environment can signal that student-student communication is valued as highly as teacher-student communication. CTP math instructors are encouraged to arrange student desks and tables so that students can easily see and respond to each other. [*See Appendix K for diagrams and commentary on typical classroom layouts and classroom layouts in CTP.*]

Students are routinely asked to pair or group themselves in order to discuss problems, but really there are almost no instances when we discourage students from collaborating. An important part of developing students' mathematical reasoning is giving them the chance to speak to one another and a skilled instructor about their mathematical ideas. Improving students' communication skills is harder to achieve in courses where computer software is a central teaching tool. Computer software can give direct instruction to students along with practice and item analyses that may point to weak skill areas, but students sitting in front of a computer may have no opportunities to explain their thinking or questions to others.

There are often several valid ways to solve math problems. When a course is moving quickly, though, an instructor can unintentionally give students the idea that there is a "right" or "best" way to do each type of math problem. This can contribute to students' discouragement and poor persistence in math classes. When students are trained to do math in this way and they see a problem but do not recognize it instantly or remember the "right" way to solve it, they can be helpless. In CTP classrooms, instructors seek out and value student descriptions of alternative solution methods. Even when an alternative solution method may appear less "efficient" than others, these methods can reveal important underlying mathematical relationships. Discussions around alternative solution methods may also reveal creative problem-solving strategies that benefit all students, and the practice of looking at problems from several directions is another element of thinking and learning like scientists.[49]

CTP math instructors are encouraged to see student errors as critically-important learning opportunities for the whole class. When this is communicated to students, they may begin to feel safe enough to make an effort even when they are unsure about how to proceed. For instructors, student errors are also a vital window into what is going on in students' minds and is a part of what makes us "assessment-centered". In classes where covering material is the driving force, however, student confusion and errors can unfortunately be seen as interruptions to the speedy flow of the lesson.

CTP instructors value the correct use of math vocabulary. Still, we permit students' informal ways of expressing mathematical ideas as we gradually press students over the semester to clarify their speech and writing to incorporate more formal and accurate math language. The only way that students will develop the spoken and written language of math is to talk and write about math. Sitting quietly and taking notes or doing computer-based drills will not tend to develop this ability. This is important for all students—native-speakers of English and English language learners alike.

[49] For more discussion on the advantages of allowing multiple solution methods in the math classroom, see How Students Learn: History, Math, and Science in the Classroom, published by the National Academy of Sciences, Chapter 5, pages 223-227, by Karen Fuson, Mindy Kalchman, and John Bransford.

College-like expectations in a nurturing environment

GED math classrooms can differ from college math classrooms in several respects. GED instructors do not assign grades to students that become a part of any transcript. This does not mean GED math instructors never give exams or require homework but many do not. Students with complex lives may have uneven attendance in GED programs which makes it difficult to scaffold learning. Teachers may respond to this by teaching isolated skills so that a day's lesson does not seem to depend on previous ones or build to later ones. Often, GED students study for successive cycles until they are judged "ready" to take the exam by the program and/or by the student.[50] Of course this differs from college environments where grades on homework and summative exams carry consequences, attendance policies can be very stringent, and content must be mastered according to the academic calendar and not at students' own pace.

CTP math classes include activities and practices that are designed to prepare students for the new expectations of a college math class. A substantial homework problem set is given in every class and students' completion is tracked. Three summative assessments are given in a CTP math semester—one after each third of the course. Because many of our students do not have a clear idea how to study for a math test, we are increasingly taking time to explicitly model strategies in this area. The emphasis we put on student communication should make it clear that our idea of assessment goes well beyond summative assessments, but these sorts of exams are a reality in college math classrooms and students need experience preparing for them.

As was described earlier, CTP math teaching almost never includes lecture. Once our students reach college classes, however, they will face lectures in some courses. To prepare students in the note-taking and other skills that are needed in that environment, we are beginning to incorporate mock math lectures where students can discuss and practice strategies for getting the most out of that teaching style.

CTP math instructors enforce a rigid system of managing student work that is designed to show students the value of being organized. Students receive a math binder at the beginning of the course, all handouts are three-hole punched, and students are expected to keep every sheet of paper dated and in chronological order. There is no textbook except for the one students build over the semester. We give attention to binder upkeep at the beginning of the course because students often need cues to keep it orderly until this becomes automatic. The advantage of a binder system over the more typical pairing of a spiral notebook for notes and a pocket folder for handouts (or no system at all) is that the binder ensures that notes and activities from the same class treating the same math topic are appropriately beside each other.

Photo by Sam Seifnourian

Bibi with her CTP math binder.

Helping students learn how to study for a math test, take notes from a lecture, and stay organized using a math binder are examples of how we try to prepare students on the academic habits they need to be successful in college. Freshmen "study skills" courses and workshops can have these goals as well, but they may present lists of habits to students divorced from content. The suggestion "Be organized" is common but it cannot help students who have no idea how to be organized in their work, or exactly why it is useful. In CTP, we ensure students stay organized by explicitly defining it and insisting they do it. Over the semester, as students see that they are able to go back and find notes on earlier discussions, they realize the value of the resource and what it took to create it.

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[50] The CUNY Adult Literacy/GED Program is notable in its academic approach to GED instruction. Examples include content-based teaching of reading and writing, the use of rich curricula in GED subject areas rather than a reliance on test-preparation materials, and learning that is scaffolded over long instructional cycles.

College study can be intense, and the significant classroom and homework demands in CTP help students to prepare for this aspect of college academics. Not counting some 6-10 hours per week of homework, CTP students attend 14 hours of reading, writing, math, and advisement sessions per week.[51] Excellent attendance is critical in CTP in part because the math curriculum is structured so that new material almost always is related to and/or relies on previously-studied ideas. Additionally, all CTP students are scheduled to take the COMPASS exams at the end of the course. In a few instances, students who are particularly weak have benefited from repeating a CTP semester, but this has only been offered to students who have shown extraordinary effort and attendance.

At the same time that CTP math classes involve intensive instruction, challenging content, and high attendance, homework, and assessment expectations, we aim to preserve the nurturing learning environment that is one of the real strengths of GED programs. This means making certain that our classrooms are safe places to raise questions and be incorrect as well as to value students' different ways of thinking about and solving problems. The learning community and advisement structures add to this environment by helping to build trusting student-teacher-advisor relationships so that students are not anonymous, and so that we can help students with outside issues that may affect their study.

[51] In addition to other changes, the re-structured College Transition Initiative (CTI) that replaces CTP in the fall 2009 semester includes a significant increase in instructional hours. CTI classes in math, reading, writing, computer skills, and “college knowledge” have expanded to 25 hours per week. Despite this increase in class time, we still require math homework twice per week from students.

The Living Curriculum

The College Transition Math Curriculum is a "living" curriculum because it has undergone significant revision every CTP semester and we expect this to continue. While I play an important role in the development and writing of the lesson materials, the entire team of CTP math instructors is involved in this process. Instructors give significant editing and other suggestions to lessons before teaching them when they are in draft form, and after teaching them in emailed reflections. In-person meetings a few times each semester give our team the chance to step back and look at the curriculum more broadly to decide if we are achieving what we planned in particular activities and over the course as a whole. For concepts that have been particularly troublesome for students, we have had much conversation leading to completely revamped teaching approaches semester-to-semester.

Dense teacher notes that are based on our experiences with CTP students accompany each activity to guide instruction. These notes suggest ways to introduce, develop, and transition between activities, point out likely trouble spots, and give guidance on managing classroom discussions. The notes serve both as a depository of our growing knowledge about how students learn and as a guide that reminds us how to plan and execute our lessons so that they embody CTP teaching and learning principles. Because conversations between the staff developer and the math team contribute a great deal to the writing and refining of these pedagogical notes, it is difficult to separate curriculum development from staff development.

The team of CTP instructors all have input into the curriculum, but once the curriculum is ready for a semester, all agree to teach the lessons in the way they are intended. This means CTP math instructors have less freedom to plan and execute lessons than in typical GED or remedial math classes. The benefit of teaching in such a structured project, though, is that we are able to have extremely focused conversations about pedagogy because we have all done the exact same activities with our students. When one of us mentions the *Best Buy* problem, we all instantly know what he/she is talking about and can focus immediately on the challenges students face in that activity.

The three CTP summative assessments (one after each third of the course) include multiple-choice as well as free-response questions. This balance gives students some practice in a multiple-choice environment where they will ultimately need to perform, but it also gives us the opportunity to see unstructured responses which can reveal more about student thinking. All CTP math classes use the same exams which are tightly aligned with the curriculum we are teaching. The instructor team also edits assessment items to be sure they are well-constructed, that they reflect the work we do in the course, that multiple-choice items have good “distractors”, and to standardize the way we assess student work on free response items.

While we have not referred to the work of the CTP math team as a “faculty inquiry group”, our collaborative work around the common CTP curriculum and assessments certainly resembles these groups as they are described in recent reports on developmental education.[52]

A number of administrators and educators who are trying to build or improve college transition programs have requested copies of the CTP math curriculum. These well-meaning educators reasonably hope that a thoughtful curriculum can be picked up and used to good ends by instructors in another program. This may be true to some

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[52] In *“The Promise of Faculty Inquiry for Teaching and Learning Basic Skills”*, a 2008 report for the Carnegie Foundation for the Advancement of Teaching, Mary Taylor Huber points to “faculty inquiry groups” at several colleges as important vehicles for improving instructional practice. In broad terms, Huber describes these groups as “teachers…looking closely and critically at student learning for the purpose of improving their own courses and programs.” Among other benefits, she writes, “As a part of the larger scholarship of teaching and learning movement, it [faculty inquiry] also involves going public with insights, experiences, and results that other educators can evaluate and build on.” The document you are reading right now is an example of how CTP math is “going public”.

degree, but instructors are unlikely to utilize new, dense materials, especially ones that approach topics in non-traditional ways, unless those materials are presented or developed alongside intensive training activities where instructors can work through activities and participate in conversations about the underlying pedagogical approaches and objectives. The challenging task of recruiting, training, and retaining capable instructors is at least as important as identifying appropriate curricula. Without thoughtful and dedicated instructors, after all, the most creative curriculum in the world is just a stack of paper.

Instructor Recruitment and Development

Photo by Sam Seifnourian

Mei and Kevin discuss a problem.

There is a shortage of adult literacy math instructors with strong content knowledge and pedagogical training around the country. This is also an issue in K-12 math teaching, but it is more acute in the adult literacy field because salaries and benefits are generally lower than in K-12 contexts. Finding a team of GED math instructors who can go beyond the GED and prepare students in the algebraic skills and reasoning needed for college is an even greater challenge. In our early work in CTP, we have built a small, capable team of math instructors. It is important to describe how we have recruited and supported the development of this team.

My principal role as the Mathematics Staff Developer for the CUNY Adult Literacy/GED Program has been to write math curricula and provide staff development for basic education and GED math teachers across 14 campus programs. In this role, I have met and worked alongside math instructors with varying abilities and interests. In these staff development activities, and despite differences in GED and CTP math content, I stress many of the same pedagogical principles that were outlined in the CTP math teaching and learning section above. Before joining CTP, our math teachers attended intensive seminars and workshops that reinforced these principles in basic education and GED-level materials. Our entire team has also been active in varying degrees over the past few years in an adult literacy math teacher collaborative, and this collaborative has a strong tradition of featuring student-centered, problem-solving approaches to math instruction.[53] CTP math teachers, then, were already connected to an unusually vibrant mixture of professional development opportunities that encouraged them to experiment and reflect on their teaching as well as to engage in a variety of pedagogical practices that are valued in CTP.

In addition to professional development the teachers received before joining the program, the CTP model for inducting math teachers is unusual and intense. Each new CTP math instructor spends a paid semester co-teaching alongside a more experienced instructor. Shortly before the start of the first CTP semester in spring 2007, Wally Rosenthal and Kevin Winkler were invited to join me in the first CTP math class as paid co-instructors. Wally and Kevin each had several years of teaching experience when they showed interest in the new course. I was the principal instructor, drafted the lessons, and directed most whole-group discussions. Wally and Kevin observed these discussions, assisted students when they were working on problems individually or in groups, designed and delivered lessons on a few topics, and met with me after each class to discuss student learning. They were strong instructors whose observations of student understanding and confusion helped to guide upcoming lessons, but at the same time they were observing my techniques for fostering communication and understanding.

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[53] The New York City Math Exchange Group (MEG) is a math teacher collaborative that was formed in 1992. MEG teachers participate in a range of activities that include doing math problems, reviewing math lessons and student work, devising and carrying out classroom-based research projects, sharing resources, and more. For more on MEG history and aims, see *"The New York City Math Exchange Group: Helping Teachers Change They Way They Teach Mathematics"* in *Focus on Basics*, Volume 4, Issue B, September 2000.

Photo by Sam Seifnourian

Joel and Christina discuss a problem.

I believe the practice of inducting new CTP instructors through a semester of co-instruction is the most effective staff development project I have ever led or participated in. After a semester working together, Wally and Kevin had the confidence to take on their own classes the following semester, and I had confidence that there was a shared understanding of CTP teaching and learning principles. Christina Masciotti, the third member of our current math team, was also inducted in this way. We see this as a rare and important opportunity for experienced instructors to work together and share ideas about pedagogy, materials, and student learning over an extended period of time.

We have used a few paid tutors in CTP math classes, but only when we are certain the tutors are able and willing to reinforce the particular pedagogical approach we use in CTP. In two out of three instances, tutors have been CTP alumni. When we employ math tutors, they attend the class sessions and work with students alongside the instructor.

We have begun to share our practices with remedial math faculty at CUNY. Our collaboration with math faculty members at LaGuardia Community College began after a leader of a remedial math initiative at the college (Project Quantum Leap, or PQL) observed a CTP math class in the fall 2008 semester. This led to a CTP presentation for PQL faculty on our approach to math pedagogy and curriculum. We have deepened our collaboration in the fall 2009 semester, including a focus on one topic from the remedial algebra syllabus. After seeing a demonstration of the CTP approach to the topic, a group of LaGuardia faculty agreed to teach (or observe a colleague teach) the same lesson to see how students would respond to a more student-centered approach.

Photo by Sam Seifnourian

Ngoc and Paul are shown working together in a fall 2008 CTP classroom. When she was a CTP student, Ngoc showed the ability to help other students without doing the work for them. Ngoc later excelled in her college pre-calculus course and now works as a CTP math tutor.

CTP math has also stretched into some New York City high schools. We have actively collaborated for the past year with *CUNY At Home in College* which has the mission of improving college access and success for high school seniors who are on track to graduate but whose test scores indicate they are likely to place into remediation (and therefore have difficulty) at CUNY. *At Home* asked CTP to provide staff development and curriculum assistance in the fall 2008 semester to 10 high school math teachers working in *At Home*-affiliated schools. Based on the available instructional time, high school teachers used a version of the CTP curriculum when the course was limited to 42 contact hours. *At Home* did not have the resources to support the co-teaching method of staff development we employ in CTP. More traditional staff development workshops were used to move teachers through student activities, especially emphasizing tasks that would likely require teachers to depart from their typical practices.

Despite offering fewer contact hours than the current CTP model, having no ability to control which teachers volunteered or who were assigned to do the work, using a less-intensive workshop model for staff development, and experiencing uneven attendance by second-semester seniors who did not need the class/credit to graduate, students in the *At Home*-affiliated high schools performed better on the CUNY math placement exams than students with similar academic profiles at the same high schools the preceding year.[54]

[54] Thirty-one percent (31%) of the *At Home* high school students passed the algebra placement exam compared with 13% of similar students in 2008 (and compared with 14% of high school students taking the exams CUNY-wide in the fall 2008 semester). Results on the pre-algebra exam were also better for *At Home* students when compared to their counterparts one year earlier (48% versus 41%).

Recommendations for GED Programs

The importance of college transition work has recently become a very popular topic in the adult literacy field. For programs looking to begin, expand, or improve math teaching for college-bound GED students, I offer the following recommendations based on my experience building CTP math.

Research college placement exams. College transition math instructors should learn about the content, format, and testing conditions of the placement exams both from local colleges and directly from exam publishers. Take the exams to experience the testing conditions and see authentic items. Clarify the precise passing scores that are needed with the testing department and ask for data showing which exam components are the most problematic for entering GED graduates. Meet with faculty and staff at the colleges to learn about opportunities that may be available for students who do not pass the exams. In some cases, colleges offer re-testing to students who attend inexpensive or free workshops.

Include significant work with functions in GED math preparation. Students should do considerable work with functions in GED math classes, especially linear functions in realistic contexts, because they touch on a large number of skills and reasoning that students need for the GED math exam. These include reasoning around graphs, data tables, equations, expressions, formulas, and written descriptions of situations. In addition to helping students pass the GED, work with functions can build students' facility and comfort with variables, expressions, equations, and functions which will help them when they must switch to a more abstract approach to algebra in a transition class and/or when they arrive at college.

Offer "algebra-for-college" instruction apart from and in addition to GED math instruction. The GED and college placement exams pose very different math challenges. Students preparing to take the GED exam need a math course focused on GED math content. For students who also intend to go to college, a separate algebra course should be offered. It is seductive to think a single curriculum might prepare students for both math demands, but in my view this is not realistic. The idea of "killing two exams with one curriculum" fails to recognize how poorly aligned the GED and college placement exams really are, or the significant time that is needed to prepare students for each. Remembering how challenging the GED math subject test is for many students, a course that additionally tries to teach the math content valued by colleges (trinomial factorization, for example) could actually delay students in reaching their first goal of earning a GED.

Provide significant instructional intensity in a college transition math course. Most GED students cannot adequately prepare for college math placement tests in a few days or weeks. If a "quick fix" was possible, pass rates on the exams and in college courses would not be as low as they are. CTP math courses have included as few as 39 instructional hours, but we have been much more satisfied with outcomes in our 72-hour math course. Our re-structured program is further lifting instructional intensity over 100 hours which may be what is necessary to help students with the weakest foundations in algebra to thrive.

Adopt the pedagogical principles of CTP math. Our experience building CTP suggests that GED programs should adopt instructional approaches that emphasize depth of understanding over coverage of many topics, develop students' conceptual understanding and not simply memorization of rules, foster student communication (rather than passive note-taking) of mathematical ideas, and that build student inquisitiveness and joy in math learning. This is valuable not only for transition math classes, but for basic education, pre-GED, and GED math teaching as well. It is effective, it improves students' confidence and persistence, and it is an interesting and engaging way for students to learn math.

Invest in the development of rich curricula that go beyond test-prep books. In order to adopt the pedagogical principles of CTP math in a transition program, instructors will need to move away from relying on test-preparation books as their core math texts. Again, this is important for math teaching at all levels. Most test-preparation books emphasize coverage rather than depth and so are not ideal for use with students who have significant math weaknesses. Resources and expertise are needed to support instructors who must find and create more engaging materials that will delve deeply into math topics for transition students.

Hire and train instructors whose content and pedagogical knowledge makes them well-suited for this work. GED and transition math instructors must have excellent math content and pedagogical knowledge to be effective. In order to embrace CTP pedagogical principles, an instructor would need to appreciate and be curious about the many ways that math topics might be introduced. We look for creative instructors who are constantly modifying their instruction, even if it takes them in directions that are quite different from the ways they themselves learned math. As a way of revealing instructors' teaching philosophies as well as their willingness to experiment in the classroom, ask the following in an interview—"*When you think back to the math teachers you had when you were in school, do you teach in a similar way? Why or why not?*"

Consider offering college transition teaching and advisement in the semester after students have earned their GEDs. With the GED behind them, students will have more time and will be more focused on preparing for entrance exams as well as completing critical application and enrollment activities. Programs should confirm that adult education funders will support this post-secondary work with GED graduates. Advocates for adult literacy funding should press for rule changes where necessary to ensure that current or new funds can be used to serve GED *graduates* and not only GED *students* preparing for college.

Adjust. Gather information on students' college math experiences that will help to shape your transition math program. Carefully review placement test results and gather student experiences and performance data in college math courses. Reviewing this information may lead transition instructors to make important changes in the content or pedagogy of the transition math class.

Recommendations for College Remedial Programs

The following recommendations are based on my understanding of the typical practices, placement mechanisms, and student populations in college remedial programs. When an individual recommendation refers to a practice that is specific to CUNY colleges, it is noted.

Promote the pedagogical principles of CTP math. CTP data suggests that GED graduates (and perhaps other underprepared students) can significantly improve their math ability when instruction emphasizes depth of understanding over coverage of many topics, develops students' conceptual understanding and not simply memorization of rules, fosters student communication (rather than passive note-taking) of mathematical ideas, and builds student inquisitiveness and joy in math learning. More typical remedial math practices such as covering long lists of topics or focusing on memorizing math rules should be reconsidered.

Invest in the development of a rich set of remedial math curricula that go beyond textbooks and syllabi. Instructors may let the textbook or their own historical approaches determine how they develop math topics each semester. The art and challenge of teaching is to constantly look beyond these sources to experiment with instruction and assessment in order to enhance student learning. Rather than limiting our concept of a curriculum to a syllabus and textbook, a richer curriculum can document the shared experience and reflections of a group of faculty as they refine their teaching over time. A curriculum in this way serves as a tool for investigating teaching and learning and is a critically-important tool for faculty development.

Intensify faculty development on and between campuses. Expertise in teaching and learning can develop when faculty (including adjunct faculty) have opportunities to meet and discuss student work, observe each others' classrooms, write and evaluate materials, and design or evaluate research alongside campus offices of institutional research. Resources must be found to make this possible at more campuses involving more faculty members. At CUNY, the challenges of remedial math instruction are present at all of the community and comprehensive colleges and there should also be regular opportunities for instructional leaders, researchers, and math faculty to come together (in person and in a regular publication) to share research, innovations, curricula, placement strategies, and other promising practices.

Create faculty leadership groups devoted to remedial mathematics. The particular challenge of remedial mathematics is so formidable and encompasses such a large fraction of students and faculty at CUNY colleges that a group of full-time math faculty members are probably needed at each campus to lead and coordinate innovations in all areas of remedial mathematics (pedagogy, curriculum, assessment, and research). Assembling a remedial math faculty leadership group at each campus could be challenging because it would require math professors to commit their time and research agenda almost exclusively to the least-prepared math students at their college. Additionally, math departments and college administrations would need to reassure potential group members that this focus on remedial students would be rewarded, for example in tenure decisions.

Widen research on prevalent and promising practices. Colleges already collect information on remedial math course retention, grades, and pass rates and they may use this data to try to measure the effectiveness of structural changes such as expanding instructional intensity, tutoring, or use of computers in a course. Despite this important work, there has been relatively little data collected or shared that would describe what remedial math instruction, curriculum development, and faculty development actually look like in practice. In a recent report published by the Carnegie Foundation, Lloyd Bond argues there is a need for increased collaboration between institutional researchers, instructional experts, and faculty to increase the research focus on teaching and student learning. I am in agreement with Bond's assertion that a bolder approach to institutional research would involve "researchers working as partners with faculty and other educators on campus to shape consequential questions about student learning, generat[ing] evidence in response to those questions, and work[ing] together toward improvements."[55] In my view, the following are areas where remedial math instruction, curriculum, staff development, and student learning might be examined at community colleges. These recommendations are only the beginning of a conversation that should ideally happen between researchers and instructors as described above.

- In-class observations could measure the extent that topics are introduced through lecture compared with more student-centered approaches. Observations could also measure the amount and types of student talk in the classroom, revealing the emphasis placed on student communication of mathematical ideas. [56]
- A review of curriculum documents could measure the degree that math departments at different campuses balance coverage versus depth. As was done at Pasadena City College, a college could experiment with remedial math sections that treat a more limited range of topics in more depth to measure impacts on student learning.[57] Curriculum documents may also reveal how much or little guidance is given to faculty on the range of pedagogical approaches that may be used to approach individual topics.
- A survey of faculty development activities could document what is available for new and continuing instructors. Do remedial math faculty meet to talk about the topics that are the most challenging for students, compare pedagogical approaches to the topics, or look together at student work? Are they involved in refining curriculum documents in ways that go beyond adding, removing, or re-ordering topics? How are new instructors inducted? Are adjunct faculty or tutors included in meaningful staff development activities? In places where faculty are routinely engaged in shared reflection on teaching practice and curriculum development, what conditions seem to make this possible?
- Measures of student learning must go beyond course grades, test scores, and retention rates. Many students who enter college underprepared in math also possess very poor "productive disposition". For students who have a history of struggling with mathematics, do college remedial math courses seem to increase students' belief that they can learn mathematics? Is there an association between students' self-efficacy and their retention? Does an active, student-centered instructional model impact students' productive disposition?

Place students in courses that provide the instruction and intensity they need. At CUNY, students fail the COMPASS algebra exam and are placed into remedial algebra courses despite possessing very different math and English abilities. CUNY students who require remedial algebra often fall into one of the following categories—(1) students who have profound algebraic weaknesses, (2) immigrant students who have strong math ability but whose weak English-language skills or unfamiliarity with U.S. math conventions and notation lead to a failing score, and (3) students who had reasonably strong algebra skills at one time but who need to restore and strengthen that knowledge. Creating different remedial algebra courses to meet students' particular needs presents an enrollment challenge, but the low pass and retention rates should challenge us to consider alternative student placement models. In order to meaningfully separate students according to instructional need, diagnostic assessments and even "math interviews" would be needed because COMPASS math scores provide so little information about students' precise math and language weaknesses. Examples of potential differentiation in remedial algebra placement are described below.

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[55] *Toward Informative Assessment and a Culture of Evidence: A Different Way of Thinking About Developmental Education* by Lloyd Bond, a report from The Carnegie Foundation for the Advancement of Teaching, 2009, page 19.
[56] Miglietti and Strange in *"Learning Styles, Classroom Environment Preferences, Teaching Styles, and Remedial Course Outcomes for Underprepared Adults at a Two-Year College"* in the Community College Review, 1988, Volume 26, No. 1, made an effort to measure how learning outcomes might compare for students sitting in "student-centered" classrooms versus students sitting in "teacher-centered" classrooms. No conclusions could be drawn from the study because none of the five math instructors were found to lead student-centered classrooms.
[57] Described in "*Technology Solutions for Developmental Math—An Overview of Current and Emerging Practices*", a 2009 report for the William and Flora Hewlett and the Bill and Melinda Gates Foundations.

- *Students who have deep algebraic weaknesses should receive a remedial algebra course with significant instructional intensity.* I believe most students who fail the COMPASS algebra exam fall into this category. These students are the most in need of instruction that embodies the CTP pedagogical principles outlined earlier and likely need substantially more instructional intensity than is typically offered in remedial algebra courses. This would improve student performance in remedial algebra as well as in subsequent credit-based math courses by building math knowledge that is more durable and transferrable. I estimate these students may need as many as 100 instructional hours to build their skills and reasoning up to adequate levels. In order for students to improve in their ability to articulate mathematical ideas and reason like mathematicians, I believe this instruction must be done by skilled instructors while tutors and/or computer software should only play a supporting role.

- *Students who are strong in math but who fail placement exams principally because they are English language learners should receive instruction tailored to their strengths and weaknesses.* COMPASS software is unable to detect whether a student misses an item due to language weakness, notational misunderstanding, or a math error; it assumes all wrong answers are math errors. It is unfortunate that English language learners (ELLs) who fail the exams despite their math strengths may be placed into remedial classes alongside students whose dominant language is English and who have much deeper math weaknesses. The numbers of immigrant students entering CUNY colleges each semester are large enough that distinct ELL remedial algebra sections could be created for those with strong math backgrounds. The algebra workshops CTP math instructors have provided to ELLs in another CUNY program (including our important focus on student communication) suggest to us that significant numbers of students may fit this profile and also that this work may be possible using fewer instructional hours than is typical in current remedial algebra courses.[58] Brief “math interviews” would be needed to determine appropriate students for these sections because neither paper nor computer exams may be able to discern the true source of student errors.

- *Students who have some algebra skills but who need to refresh or strengthen them should be given a course designed for this purpose.* If all students who fail the COMPASS algebra exam were given a follow-up diagnostic exam and/or math interview, it could be used to separate this group from students who have more fundamental weaknesses.[59] A separate remedial algebra course for stronger students could perhaps include the ELL student population described above or could also be imagined as a distinct, third offering. Instructional hours needed in this course could be lower than what was described for the weakest students—perhaps similar to current levels. Even though the course could move more quickly than the lowest-level offering, it would benefit from adopting CTP pedagogical practices, especially our emphasis on depth of understanding over coverage. Another benefit of adding a higher-level remedial algebra course would be that students in the lowest-level course would have a slightly different course to move into if they were not able to place out of remedial algebra in their first attempt. Currently, the weakest students move through a curriculum at too fast a pace and then repeat the same experience with very poor results.

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[58] CTP math instructors have been providing math review sessions to some ELL students in the CUNY Language Immersion Program (CLIP) for close to three years. The CUNY Language Immersion Program (CLIP) is an intensive English-language program offered on nine CUNY campuses as a low-cost alternative to remedial ESL courses for immigrant students who have been admitted to a CUNY college and who failed the writing or reading placement tests. When students complete their stay in CLIP and are preparing to fully enter the college, CTP math instructors lead math workshops for those who failed the algebra section of the COMPASS. Despite brief math interventions, student pass rates are surprisingly strong. Some of this success is certainly due to their improved English language skills, but the intervention appears to also play an important role in helping students connect their previous math understandings to an English-speaking classroom. In our interactions with these students, we have discovered that it is possible in a few moments to determine which of the students have strong math backgrounds that were not captured by the COMPASS exams. This has led me to the belief that “math interviews” could identify students who would benefit from ELL remedial math sections.

[59] I do not recommend using COMPASS algebra scores to divide the students because I do not have confidence that the scoring is sensitive enough to identify students’ relative algebra strengths, especially knowing that COMPASS software may end an exam and deliver scores to students after as few as ten questions.

Place students needing remedial math instruction into learning communities where possible. The transformation of CTP into a learning-community model made us stronger than the sum of our academic and advisement parts. The cohesiveness that developed between students and staff over the semester certainly contributed to student persistence and success. Learning communities will likely be the most successful when staffed by full-time instructors and advisors so that they have the time for meaningful collaboration.

Partner with high school and GED programs in preparing students for college success. Partnerships exist and are strengthening between The City University of New York and the New York City Department of Education to improve high school graduates' transition into CUNY colleges. In addition to these initiatives, math faculty can reach out to nearby "feeder" high schools and GED programs to share information on the math (and other) challenges students are facing as they move into college. Exchanges between college math faculty, high school, and GED math teachers can help to clarify the similarities and differences between math content on the various exams and can also address pedagogical issues. Understanding more about college math challenges might also lead high school administrators to make choices that will help their students, such as ensuring that more college-bound seniors take a fourth year of mathematics where this is not already a requirement. CUNY projects including *At Home in College* and *Looking Both Ways* have advocated these sorts of high school-college instructor collaborations and have seen them not as solitary encounters but ideally as conversations that extend over time.

Appendix A – GED Score Information for CTP Students

GED scores for 62 CTP students from three intensive cohorts are not exceptional when compared with typical GED graduates who enter CUNY.[60] Mean and median subject test scores are actually somewhat lower in writing and math for CTP students, the two areas that are normally the most problematic for GED graduates as they transition into college.

GED Test Score Information for Three CTP Student Cohorts Compared with All GED Graduates Entering CUNY[61]

	Mean/Median CTP	Mean/Median CUNY
Writing	484 / 465	497 / 480
Social Science	515 / 510	518 / 510
Science	504 / 500	512 / 500
Reading	530 / 500	525 / 500
Math	484 / 470	496 / 480
Total	2517 / 2470	2547 / 2470

[60] GED scores were collected for 62 of 66 students who attended at least one CTP class in the fall 2008 and spring 2009 intensive CTP classes at LaGuardia Community College and Borough of Manhattan Community College. Three of the missing students dropped from CTP before their GED information could be collected. The fourth student had a Canadian high school diploma but no GED.

[61] In order to pass the GED, students must score at least 410 on each subject test and 2250 in total. This requires an average score of 450 across all five tests. Twenty-one (21) of 62 CTP students measured here earned a writing score below 450 and seventeen (17) earned below 450 in math. These students needed to make up for their weak areas by scoring higher than 450 in other subject tests. Thirteen students (21%) earned extremely low passing math scores (410 or 420). The CUNY-wide figures for GED graduates are taken from *College Readiness of New York City's GED Recipients*, CUNY Office of Institutional Research and Assessment, 2008, page 5 of the data tables section. These statistics were calculated based on available scores for GED graduates entering CUNY in the 2001-2002, 2004-2005, and 2006-2007 academic years.

Appendix A

GED Subject Test Scores for CTP Students, Fall 2008 and Spring 2009							
Writing	**Soc Sci**	**Science**	**Reading**	**Math**	**Total**	**Name**	**Site/Semester**
440	490	440	460	420	2250	Paul	LaGuardia/fall 08
480	430	440	440	480	2270	Miaona	LaGuardia/fall 08
480	450	440	500	420	2290	Alie	LaGuardia/sp 09
440	480	450	410	510	2290	Maria	BMCC/spring 09
410	480	530	430	450	2300	Linh	LaGuardia/fall 08
450	490	430	520	420	2310	Bibi	LaGuardia/sp 09
430	460	460	430	530	2310	Mei	BMCC/spring 09
510	460	470	430	450	2320	Tishanna	BMCC/spring 09
440	460	510	440	470	2320	Rafael	LaGuardia/sp 09
450	470	470	500	440	2330	Amparo	BMCC/spring 09
430	470	430	480	520	2330	Alex	LaGuardia/fall 08
420	470	440	460	540	2330	Chime	LaGuardia/sp 09
410	550	520	440	420	2340	Mario	LaGuardia/fall 08
430	510	470	460	470	2340	Daniel	LaGuardia/sp 09
450	460	500	440	500	2350	Elsa	LaGuardia/sp 09
440	470	470	470	500	2350	Araceli	LaGuardia/fall 08
430	490	480	500	470	2370	Tenzin	LaGuardia/fall 08
510	440	430	510	480	2370	Ramatoulaye	LaGuardia/sp 09
430	480	510	440	520	2380	Luis	LaGuardia/sp 09
500	530	450	500	410	2390	Aaza	LaGuardia/fall 08
490	480	490	480	450	2390	Teresa	LaGuardia/sp 09
420	530	510	480	450	2390	Luis	LaGuardia/fall 08
440	470	490	480	510	2390	Angel	LaGuardia/sp 09
450	480	520	480	470	2400	Dario	LaGuardia/sp 09
450	500	500	540	440	2430	Susie	LaGuardia/fall 08
430	530	500	520	450	2430	Ricardo	LaGuardia/sp 09
430	510	500	540	450	2430	Rose	LaGuardia/fall 08
450	500	490	520	470	2430	Johana	LaGuardia/fall 08
480	520	510	480	460	2450	Edy	LaGuardia/fall 08
460	500	490	520	480	2450	Fatmir	LaGuardia/sp 09
510	560	500	480	410	2460	Eugina	BMCC/spring 09
540	470	480	490	500	2480	Emanuel	BMCC/spring 09
480	530	480	600	410	2500	Patricia	LaGuardia/sp 09
460	570	550	520	420	2520	Robert	LaGuardia/fall 08
440	530	540	500	510	2520	Kazi	LaGuardia/fall 08
530	540	470	570	420	2530	Roxane	LaGuardia/fall 08
460	510	480	570	510	2530	Francisco	LaGuardia/sp 09
590	570	520	440	420	2540	Ricardo	BMCC/spring 09
510	520	480	600	430	2540	David	BMCC/spring 09
520	440	480	560	540	2540	Argenis	LaGuardia/sp 09

480	520	560	480	510	2550	Toribio	LaGuardia/fall 08
600	530	470	490	460	2550	Andrea	LaGuardia/sp 09
440	570	540	480	530	2560	Hilda	LaGuardia/sp 09
550	480	520	610	440	2600	Brenda	LaGuardia/fall 08
530	490	500	630	450	2600	Jacqueline	LaGuardia/fall 08
570	550	510	560	420	2610	Alfred	BMCC/spring 09
420	490	600	610	510	2630	Joel	LaGuardia/sp 09
550	490	480	600	520	2640	Maggie	BMCC/spring 09
420	510	560	620	530	2640	Smith	LaGuardia/fall 08
470	560	490	670	460	2650	Johnny	LaGuardia/sp 09
610	570	470	620	410	2680	Jenny	LaGuardia/fall 08
470	600	610	490	510	2680	Gregory	BMCC/spring 09
440	550	530	520	640	2680	Tenzin	LaGuardia/sp 09
490	590	560	590	490	2720	Tsering	BMCC/spring 09
480	520	680	470	600	2750	Ngoc	LaGuardia/fall 08
570	530	480	590	590	2760	Shardise	BMCC/spring 09
520	530	560	600	570	2780	Suchuntra	LaGuardia/sp 09
460	660	530	760	420	2830	Giselle	LaGuardia/sp 09
550	590	560	670	480	2850	Gabriela	LaGuardia/fall 08
570	590	560	800	480	3000	Denneileia	BMCC/spring 09
570	600	550	710	680	3110	Miguel	LaGuardia/fall 08
700	610	630	660	710	3310	Wilson	LaGuardia/fall 08

Appendix B—CTP Retention, Application, and Testing Data

Rates of Completion, Applications to CUNY, and Placement Testing for Three CTP Cohorts[62]	
Number of students who attended at least one class session	66
Number of students who completed the course	55
Completion rate for all starters	83.3%
Completion rate for students who attended at least two weeks of the course	87.3%
Percent of completers who applied to CUNY	96.4%
Percent of completers who took the CUNY placement tests	94.5%

[62] The data for three CTP intensive cohorts was drawn from the fall 2008 and spring 2009 classes at LaGuardia Community College and the spring 2009 class at Borough of Manhattan Community College.

Appendix C – CTP Math Assessment Results

The chart below shows pre- and post-test scores for CTP math students in the fall 2008 and spring 2009 intensive cohorts. Because the assessments were revised between the fall and spring semesters, they have different total possible scores. Consequently, raw scores were turned into percentages before an average could be taken for all three groups.

Total Pts	Pre-Test Raw Score	Pre-Test %	Post-Test Raw Score	Post-Test %	Name	Site/Semester
45	3	6.7	29.75	66.11	Aaza	LaGuardia/fall 08
45	6.25	13.9	34.50	76.67	Alex	LaGuardia/fall 08
45	24	53.3	44.00	97.78	Araceli	LaGuardia/fall 08
45	7.25	16.1	29.50	65.56	Brenda	LaGuardia/fall 08
45	7.5	16.7	41.00	91.11	Edy	LaGuardia/fall 08
45	19.25	42.8	42.50	94.44	Elsa	LaGuardia/fall 08
45	22.25	49.4	37.25	82.78	Gabriela	LaGuardia/fall 08
45	3.75	8.3	7.00	15.56	Inez	LaGuardia/fall 08
45	2.5	5.6	27.75	61.67	Jacqueline	LaGuardia/fall 08
45	5.75	12.8			Jenny	LaGuardia/fall 08
45	6.25	13.9	23.25	51.67	Johana	LaGuardia/fall 08
45	27.5	61.1	42.00	93.33	Kazi	LaGuardia/fall 08
45	30.75	68.3	39.50	87.78	Linh	LaGuardia/fall 08
45	2.5	5.6	31.25	69.44	Luis	LaGuardia/fall 08
45	13.75	30.6			Luis	LaGuardia/fall 08
45	7	15.6	39.75	88.33	Mario	LaGuardia/fall 08
45	2.5	5.6			Melisa	LaGuardia/fall 08
45	13	28.9	44.50	98.89	Miaona	LaGuardia/fall 08
45	23.5	52.2	43.50	96.67	Miguel	LaGuardia/fall 08
45	32.75	72.8	45.00	100.00	Ngoc	LaGuardia/fall 08
45	19.25	42.8	34.75	77.22	Paul	LaGuardia/fall 08
45	4.75	10.6			Priscilla	LaGuardia/fall 08
45	8.5	18.9	33.75	75.00	Robert	LaGuardia/fall 08
45	5.25	11.7	35.25	78.33	Rose	LaGuardia/fall 08
45	9.25	20.6	34.50	76.67	Roxane	LaGuardia/fall 08
45	6.25	13.9	33.25	73.89	Smith	LaGuardia/fall 08
45	14.75	32.8	27.50	61.11	Susie	LaGuardia/fall 08
45	15.5	34.4	42.50	94.44	Tenzin	LaGuardia/fall 08
45	10.75	23.9	44.00	97.78	Toribio	LaGuardia/fall 08
45	28	62.2	43.00	95.56	Wilson	LaGuardia/fall 08
45			14.50	32.22	Patricia	LaGuardia/fall 08

Appendix C

Total Pts	Pre-Test Raw Score	Pre-Test %	Post-Test Raw Score	Post-Test %	Name	Site/Semester
55	5	9.1	35.00	63.64	Alfred	BMCC/spring 09
55	26.75	48.6	49.75	90.45	Amparo	BMCC/spring 09
55	12	21.8			David	BMCC/spring 09
55	22.5	40.9	55	100.00	Denneileia	BMCC/spring 09
55	5.5	10.0	49.25	89.55	Emanuel	BMCC/spring 09
55	5				Eugenia	BMCC/spring 09
55	14.5	26.4	45.5	82.73	Greg	BMCC/spring 09
55	32	58.2	51.5	93.64	Maggie	BMCC/spring 09
55	25	45.5	53	96.36	Maria	BMCC/spring 09
55	30.75	55.9	55	100.00	Mei	BMCC/spring 09
55	13.75	25.0	41.5	75.45	Ricardo	BMCC/spring 09
55	38.5	70.0	51.75	94.09	Shardise	BMCC/spring 09
55	7.25	13.2	45	81.82	Tishanna	BMCC/spring 09
55	6.5	11.8	53	96.36	Tsering	BMCC/spring 09
55	10	18.2	41	74.55	Bibi	LaGuardia/sp 09
55	22.5	40.9	39	70.91	Alie	LaGuardia/sp 09
55	15.25	27.7			Johnny	LaGuardia/sp 09
55	15.75	28.6			Rafael	LaGuardia/sp 09
55	20	36.4	45.50	82.73	Luis	LaGuardia/sp 09
55	18	32.7	35.75	65.00	Patricia	LaGuardia/sp 09
55	30.5	55.5	49.75	90.45	Chime	LaGuardia/sp 09
55	23.5	42.7	52	94.55	Suchuntra	LaGuardia/sp 09
55	35.75	65.0	54	98.18	Tenzin	LaGuardia/sp 09
55	5	9.1			Teresa	LaGuardia/sp 09
55	10.25	18.6	48	87.27	Fatmir	LaGuardia/sp 09
55	6.5	11.8	45	81.82	Ricardo	LaGuardia/sp 09
55	15.25	27.7	36.75	66.82	Daniel	LaGuardia/sp 09
55	9.75	17.7	37.5	68.18	Joel	LaGuardia/sp 09
55	29.5	53.6	46	83.64	Hilda	LaGuardia/sp 09
55	9	16.4			Francisco	LaGuardia/sp 09
55	21.25	38.6			Argenis	LaGuardia/sp 09
55	25.5	46.4	51	92.73	Ramatoulaye	LaGuardia/sp 09
55	25	45.5	47.25	85.91	Dario	LaGuardia/sp 09
55	9.75	17.7	38.25	69.55	Angel	LaGuardia/sp 09
Mean percentages		**30.4%**		**81.04%**		All three cohorts

Appendix D—CUNY Placement Test Results for CTP Students

Initial CUNY Basic Skills Exam Scores for CTP Students
Entering Certificate, Associate's, and Bachelor's Programs, fall 2008 and spring 2009

Math 1[63]	Math 2[64]	Math 3[65]	Math 4[66]	Reading[67]	Writing[68]	Name	Site/Semester
39	19			90	7	Jacqueline	LaGuardia/fall 2008
51	21			81	6	Brenda	LaGuardia/fall 2008
36	22			64	6	Susie	LaGuardia/fall 2008
56	22			89	8	Gabriela	LaGuardia/fall 2008
24	23			52	6	Mario	LaGuardia/fall 2008
17	24			78	8	Johana	LaGuardia/fall 2008
33	24			76	8	Roxane	LaGuardia/fall 2008
23	25			83	6	Paul	LaGuardia/fall 2008
34	27			90	6	Rose	LaGuardia/fall 2008
38	28			61	8	Miaona	LaGuardia/fall 2008
32	29			79	7	Robert	LaGuardia/fall 2008
89	29			98	7	Smith	LaGuardia/fall 2008
53	34	51	28	89	8	Toribio	LaGuardia/fall 2008
60	36	47	30	68	8	Tenzin	LaGuardia/fall 2008
61	36	31	22	80	6	Edy	LaGuardia/fall 2008
54	37	40	22	87	7	Araceli	LaGuardia/fall 2008
61	58	41	27	73	4	Linh	LaGuardia/fall 2008
96	81	77	22	89	8	Kazi	LaGuardia/fall 2008
95	86	55	16	98	7	Wilson	LaGuardia/fall 2008
98	97	62	45	92	8	Ngoc	LaGuardia/fall 2008
60	25			68	6	Alex	LaGuardia/fall 2008
26	20			84	6	Luis	LaGuardia/fall 2008
53	22			82	6	David	BMCC/spring 2009
40	24			62	8	Tishanna	BMCC/spring 2009
24	28			73	6	Emanuel	BMCC/spring 2009
42	28			72	6	Amparo	BMCC/spring 2009
47	32	17		78	8	Maria	BMCC/spring 2009
30	34	15		75	10	Ricardo	BMCC/spring 2009
65	35	36	19	75	8	Tsering	BMCC/spring 2009

63 At most CUNY community and comprehensive colleges, students need a minimum of 30 to pass this exam.
64 At most CUNY community and comprehensive colleges, students need a minimum of 30 to pass this exam.
65 When a student scores 30 or higher on COMPASS math exams one and two (pre-algebra and algebra), he or she is considered proficient and is automatically given problems from a third and possibly fourth exam to help the college determine the student's appropriate placement in a credit-bearing math course. In a few instances, the scores on the 3rd and 4th exams were not available but this is not a consequential part of the data set for the analyses done in this paper.
66 *Ibid.*
67 Students need a minimum of 70 on the reading exam to be considered proficient.
68 Students need a minimum of 7 on the writing exam to be considered proficient.

Appendix D

Exempt	35	21		99	8	Miguel	BMCC/spring 2009
88	46	36	19	86	8	Maggie	BMCC/spring 2009
60	47	43	32	82	6	Shardise	BMCC/spring 2009
75	53	42	36	91	10	Denneileia	BMCC/spring 2009
77	68	50	16	58	6	Mei	BMCC/spring 2009
24	21			84	8	Alfred	BMCC/spring 2009
68	43			84	8	Gregory	BMCC/spring 2009
43	19			72	8	Ricardo	LaGuardia/spr 2009
55	20			79	6	Joel	LaGuardia/spr 2009
47	22			66	6	Daniel	LaGuardia/spr 2009
28	23			77	8	Patricia	LaGuardia/spr 2009
30	23			57	7	Fatmir	LaGuardia/spr 2009
30	27			82	8	Angel	LaGuardia/spr 2009
62	28			81	8	Bibi	LaGuardia/spr 2009
33	32	19		57	8	Alie	LaGuardia/spr 2009
50	36	40	22	58	8	Ramatoulaye	LaGuardia/spr 2009
31	38	20		79	8	Dario	LaGuardia/spr 2009
38	39	22		39	7	Luis	LaGuardia/spr 2009
46	48	42	19	80	8	Suchuntra	LaGuardia/spr 2009
66	55	47	18	65	8	Elsa	LaGuardia/spr 2009
88	62	79	31	82	8	Tenzin	LaGuardia/spr 2009
73	67	68	16	70	8	Chime	LaGuardia/spr 2009
Exempt	Exempt			87	8	Hilda	LaGuardia/spr 2009

Appendix E – CTP Math Assessment Data for Those Who Ultimately Passed COMPASS Algebra

	CTP Pre-Test %	CTP Post-Test %
Tsering	11.82	96.36
Edy	16.67	91.11
Toribio	23.89	97.78
Ricardo	25.00	75.45
Greg	26.36	82.73
Tenzin	34.44	94.44
Luis	36.36	82.73
Alie	40.91	70.91
Denneileia	40.91	100
Suchuntra	42.73	94.55
Elsa	42.78	94.44
Maria	45.45	96.36
Dario	45.45	85.91
Ramatoulaye	46.36	92.73
Miguel	52.22	96.67
Araceli	53.33	97.78
Chime	55.45	90.45
Mei	55.91	100
Maggie	58.18	93.64
Kazi	61.11	93.33
Wilson	62.22	95.56
Tenzin	65.00	94.44
Linh	68.33	87.78
Shardise	70.00	94.09
Ngoc	72.78	100
Average	**46.15**	**91.97**

Appendix F – A Math Rule As the Endpoint, Not the Starting Point

When doing work with exponents in secondary and college remedial math classrooms, multiplying terms with like bases is a common topic.

Multiply. $x^3 \cdot x^5$

Three ways of presenting this idea are described below. The first two are common in traditional math classrooms and feature the instructor demonstrating a rule that students are expected to follow. The third approach is the one that we take in CTP and emphasizes student exploration of the underlying mathematics for a considerable period of time before any rule emerges.

Method #1—A highly-abstract presentation

The instructor begins by writing the rule on the board.

$$x^a \cdot x^b = x^{a+b}$$

The instructor continues by demonstrating how the rule works with numerical exponents.

$$\begin{aligned} x^3 \cdot x^5 &= x^{3+5} \\ &= x^8 \end{aligned}$$

This presentation can confuse students for a number of reasons. First, many students are not skilled interpreting variables that are used to generalize relationships. This example will be particularly challenging because variables appear in an unusual place—as exponents. Second, a multiplication sign on the left side of the equation has disappeared and addition has appeared on the right side. This is counter-intuitive for students because they may (rightly) have the idea that exponents represent repeated multiplication. With no “back-up” for why this rule is true, it is often forgotten or misused.

Method #2—More justification but still an instructor-centered presentation

An instructor may already know that students have difficulty with this exponent rule. As a result, the instructor may want to show students why the exponent rule “works” using this demonstration.

$$\begin{aligned} x^3 \cdot x^5 &= x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \\ &= x^8 \end{aligned}$$

In this approach, the instructor will likely describe the expansion and emphasize why the product is x^8 and not x^{15}, finally concluding that we must add exponents when we multiply terms with like bases.

Certainly method #2 is preferred to method #1 for weak math students because more of the underlying mathematics is revealed. Nevertheless, this second presentation has several weaknesses. Often because of time pressures, the instructor will provide just one example before stating the more general rule. Students in this instance miss an opportunity to see several examples and recognize the pattern and rule themselves. And because the instructor (rather than the students) did the opening example, students may not have enough experience doing expansions to be able to resort to that method in case they forget the rule later. Instructors also do not typically encourage students to do expansions in their own work. Once the rule has been stated, the focus tends to shift completely to the rule over strategies that are seen as less efficient.

Method #3—The CTP approach

CTP instructors approach this topic by asking students to observe the following and decide if it is a true or false equation.

$$x^3 \cdot x^5 \stackrel{?}{=} x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

Conversations about this possibly-true equation are often lively, involve many students, and give students the opportunity to articulate their reasoning, as well as to justify and evaluate each other's assertions. Through the discussion moderated by the instructor, students ultimately will agree that it is a true equation. CTP instructors can then introduce a new vocabulary word—*expansion*—to describe the process of explicitly writing the "hidden" multiplication in an expression.

After the expansion has been confirmed as equivalent to the starting expression, students are asked if the right side of the equation can be simplified. Students arrive at x^8.

Over the course of several more problems, the expansions are what we emphasize. After students gain facility with elementary expansions, we may present them with a problem such as $x^{100} \cdot x^5$. Without us having to say a word, this problem pushes students to try to visualize the expansion in their minds to save the time that would be needed to write it. Students will hopefully imagine a row of 100 *x*'s followed by 5 more—they are thinking through (and we will question them about) the meaning of the expression rather than trying to apply a rule they may not fully understand. After several examples, some students will come to use the rule, but they do so based on problems they have done and that they can visualize. Some students will continue to use the expansion method for some time, and to us this is perfectly appropriate.

Some may claim that students who already "know" the exponent rule do not need this exposition. We disagree. For students who enter CTP with the rule memorized, our work with the expansion can add important justification and depth to their understanding. For weaker students who have much confusion with this and other rules, our focus on expansion builds a needed foundation of understanding. The best part of this work, though, is that it involves students actively in their learning much more than in either of the previous instructor-centered approaches.

Because they reveal underlying math relationships, we use expansions later in the course to introduce several concepts including multiplying terms, dividing terms, and factoring expressions.

Appendix G – Introducing Slope Without Lecture

Slope is an enormously rich concept. It describes how variables change in relation to one another, it can be found in tables of values, graphs, equations, and written descriptions of situations, and it can help us to describe and make predictions about linear and other relationships that exist in the world. Unfortunately, students in middle school, high school, and GED classrooms may explore slope in narrow ways, and in some cases may focus almost exclusively on slope as it appears in the following equations:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad y = mx + b$$

These equations are important, but students who have watched instructors demonstrate how to use them year after year often do not master them. In particular, students may have great difficulty using the formula on the left because it requires proficiency in three areas that are frequently troublesome for students—signed numbers, subtraction, and fractions.

By the time they enter college remedial math classes, many students have very weak understanding of slope. Even hearing the word “slope” may bring a grimace to students’ faces, reflecting widespread frustration and boredom with the way the topic has been presented numerous times before. In CTP, we want to explore the concept deeply while also presenting it in fresh ways that will make it interesting to learn and teach. In this appendix, we will only have space to describe the introduction of slope as an example of how we avoid lecture wherever possible in favor of developing new ideas using as much student involvement as possible.

Slope appears prominently in nine out of 23 functions activities in the current CTP math curriculum. Before we begin any work with slope, students study realistic and abstract function equations, tables of values, the “three views” of a function, function solutions, function equations and tables of values on the graphing calculator, and do some inductive work comparing function equations to the types of graphs that are created (linear vs. quadratic vs. cubic). We do not speak about slope in any of these early activities.

One of the important things we do in our introduction to slope is to avoid using the word “slope” throughout the entire first lesson. The word can trigger student discomfort as well as formulas that are poorly-remembered or understood. Instead, we give students a handout with the two function tables seen at the right (except the outputs are missing). After students complete the outputs, we ask them the following:

$y = 2x + 1$	
x	***y***
0	1
1	3
2	5
3	7
4	9
5	11
6	13

$y = 2x + 4$	
x	***y***
0	4
1	6
2	8
3	10
4	12
5	14
6	16

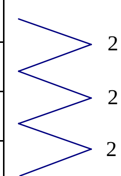
“What do you notice that makes these function equations or tables of values similar or different?”

The instructor now has the critical task of managing this discussion, pushing students to make clear, precise statements and searching for confirmation or clarification from as many students as possible. Common student observations include the following:

- *"They have the same inputs."*
- *"The outputs are different."*
- *"Both say 2x in the rules/equations."*
- *"The inputs grow by 1."* (We will look for a student who can use "consecutive" here because we just encountered the word in the previous lesson and because it is a useful part of the coming definition of "rate of change".)
- *"The outputs are odd in the first function and even in the second."*
- *"For the same inputs, the outputs on the right are 3 larger than the ones on the left."* (We will also look for a student who can connect this to the function rules.)
- *"The outputs grow by 2 in both functions."*

This last observation always arises at some point in the conversation. After hearing and getting confirmation from other students, we mark the differences between outputs on the board in the following way (and ask students to do the same) before continuing the discussion.

$y = 2x + 1$	
x	***y***
0	1
1	3
2	5
3	7
4	9
5	11
6	13



It is at this point that the instructor needs to name what students have already noticed about the function outputs. We simply tell students (there is no way for them to discover this) that the constant difference in the outputs (when the inputs are consecutive as students already noted in the discussion) is known as the *rate of change*.

"Rate of change" is a friendlier term than slope here because it describes quite literally what we are measuring when looking at the table of values. Something that is often glossed over but that we focus on overtly in CTP is that the rate of change may appear as a single number (2 in this case) even though it simultaneously describes change in two variables:

As the inputs increase by one, the outputs increase by two.

To deepen understanding of this concept (identified by students and named by the instructor), we ask students to identify the rate of change across a series of increasingly complex function tables including negative and decimal rates of change and functions in realistic contexts. After identifying and describing several rates of change, students are guided to discover the relationship between the rate of change and the coefficient of *x* in the function equation. This is challenging for students because our approach to rate of change involves recognizing difference (addition/subtraction) in inputs and outputs while the coefficient of *x* represents a multiplication relationship. We do not shy away from this but make a real effort to guide students to understand why these numbers are related. In all of the work we do in this first rate of change activity, we focus students on describing the rate of change both as a single number and in oral and written sentences that articulate the change that is happening in the input and output variables.

What is interesting about this approach is that we rarely encounter a CTP student who connects rate of change to slope on their own. This demonstrates how poor students' understanding of slope really is. It is not until the second lesson that we say the word “slope”, and we do this very directly—*“Rate of change is also known as slope.”* Students can show a lot of surprise at this.

This is just the beginning of a series of scaffolded slope activities that of course must consider non-integer slopes and the formulas that were mentioned earlier, but we have found that students feel confident working with this challenging topic after an introduction that is rooted in students’ own mathematical observations.

Appendix H – Context can be the foundation for more abstract ideas

Remedial algebra courses often introduce functions with a definition that distinguishes functions from relations along with a demonstration of the "vertical line test". Function notation may be introduced before students have mastered less formal notation. Functions in real or realistic contexts usually appear at the end of more abstract presentations, if at all.

Rather than ignoring context or relegating it to the fringes of the conversation, CTP math instructors use realistic contexts as a central way of building new student skills and reasoning about functions. We see several advantages to working with realistic functions, even though we know that the COMPASS algebra exam typically does not emphasize them.

- Realistic contexts are an excellent way to connect students' current understandings to new, more abstract, ideas.
- Realistic contexts help to show students that algebra can be used to powerfully describe and predict features of the social and physical world.
- Realistic contexts give us rich opportunities to build students' math communication skills. CTP students are expected to describe functional relationships and the meaning of inputs, outputs, slopes, and *y*-intercepts in realistic settings both orally in and in writing.
- A balance of realistic contexts and abstraction helps keep CTP lessons varied and vibrant.

Students see the following problem in the first CTP lesson.

Best Buy Commissions

You take a job at *Best Buy* selling digital cameras. Your base pay is $150 per week. For each digital camera you sell, you earn an additional $18.

Complete the table.

Digital Cameras You Sell in One Week	Your Weekly Pay
0	
1	
2	
10	
16	
	$366
	$492
	$546

The *Best Buy* problem is an example of a task that many students—including remedial math students—can solve using the knowledge they already bring to the classroom. How many remedial math students would have more difficulty solving the same problem presented in the following way?

$$\text{Given } f(x) = 18x + 150, \ f(16) = ?$$

One goal over the course, then, is to move students from being able to solve the *Best Buy* problem to solve problems that include more formal function notation.

After students have completed the table of values for the *Best Buy* problem, we pay particular attention to the methods they used to determine the missing inputs. Some use inverse operations, some use guess-and-check, and others use guess-and-check after looking at the previous output values to decide a good starting guess. We encourage students to articulate these different strategies in the very first class session, and students learn quickly that a variety of solution methods are valued in CTP.

Once the table has been completed, we bring students back to talking about how they calculated the missing outputs for the first five inputs. Students typically describe the repeated series of operations (multiply by 18 and add 150) for the inputs 2, 10, and 16. The discussion of whether those same operations are used for the inputs 0 and 1 is valuable because this initially will not be apparent to all students. Finally, we tell students that this is an example of a function, give them a brief definition in relatively informal language, and have them record the function rule they have already employed in words:

Multiply by 18 then add 150.

Using a series of function tables and rules over the next two classes, we move students from function rules that use words to rules written as equations. See the examples.

Input	Rule: Multiply by 20 then subtract 5.	Output
1		
2		
5		
		75
		135

We often include problems that require students to calculate missing inputs because these are more challenging (and interesting) for the students than simply calculating missing outputs. We do some teaching around inverse operations as a solution method but ultimately allow students to use whatever strategy they wish as long as they do not use a calculator.

Input	Output = Input x 3 + 10	Output
0		
10		
12		
		61
		121

Moving to an equation format for the function rule can be tricky, even when we use “input” and “output” before moving to single letters as variables. Notice that “Output” appears here at the beginning of the rule. We write the rule in this fashion because we are building towards the convention of writing functions in $y =$ format.

Input	Output = Input x 2 + .75	Output
1		
2		
3.5		
4.5		
		30.75
		11.75

Whenever we do work with number in another section of the class, we will immediately introduce the new concept into our function rules. Decimals are incorporated here, as are signed numbers and exponents when appropriate.

After students have worked with a series of problems that include function rules written as equations, we re-introduce context and ask students to make sense of another rule related to our first problem.

The Digital Source

A store called *The Digital Source* is trying to lure you away from working at *Best Buy*. The manager at *The Digital Source* comes to you with the following proposal:

The Digital Source

"If you come and work for me, I will pay you according to the following function.

$$Output = Input \times 8 + 200$$

In this function, the input is the number of digital cameras you sell in one week, and the output is your weekly pay in dollars."

<u>Should you take this job offer?</u> Use a separate sheet of notebook paper to write your response. Include calculations and any supporting information that you feel will clarify your reasoning for the reader. Assume the person reading your work has not seen any information about the two stores before.

What makes this writing assignment interesting is that there is more than one correct answer. Students need to defend a preference for one store or the other using the functions and the student's own assumptions about how hard it is to sell cameras or they may also conclude that "it depends" and explain why that is. After students submit their work, we often ask them to revise and re-submit their work so that their assertions are clear and supported by evidence.

After *The Digital Source*, we resume our progression towards more formal notation. When we introduce rules using *x/y* format, we push students to connect the new format to the old by asking them to re-write rules using words.

$y = 5x + 1$	
x	***y***
0	
1	
2	
3	
	56
	101

Translate this function rule into words:

Output =

Students begin with a problem (*Best Buy*) that they can solve without knowing what a function is at all. After formalizing what they already can do we guide them to write function rules in words, using equations and *x/y* notation, and eventually to use the function notation shown earlier. Throughout this progression, students must contend with both abstract and contextualized functions that give them opportunities to solidify their understanding before moving to the next, more formal, level. Realistic contexts are also used in many other areas of the CTP curriculum and are the central way we introduce systems of equations, parallel lines, and composite functions.

Appendix I – Number illuminates the distributive property

Before giving any introduction to the concept or even mentioning the words "distributive property", we give CTP students the following direction and tasks.

Answer the following problems *using only mental math*. This means no pencils, paper, or calculator.

1. In preparation for a seminar, a seminar leader buys new binders for eight participants. Binders cost $3.10 each. What is the total cost of the binders before any taxes are added?

2. Pizza Amore charges $12 for a medium pizza, plus $1.05 for each extra topping. You decide to order a medium pizza with six toppings. How much does the pizza cost before any taxes are added?

Most students do the calculations by separating the dollars and cents in their minds and then multiplying them each by eight in the first example and by six in the second example. In the follow-up discussion, CTP instructors work together with students to formalize this process.

Step one—dollars and cents are separated.

Step two—multiplication happens separately.

$$8 \cdot 3.10 = 8(3.00 + .10) = 8 \cdot 3.00 + 8 \cdot .10$$
$$= 24 + .80$$
$$= \$24.80$$

Step three—the products are combined.

What is most interesting about this is the move from "step one" to "step two". Most students have been subjected to enough drilling about the order of operations that they believe operations inside parentheses must be done before ones that are outside of parentheses. What they have done to solve the binder and pizza problems subverts that entirely. Rather than doing addition first, multiplication happens first. In fact, we emphasize this aspect of the distributive property in a few CTP activities—that the property gives us a way of doing things that does not seem to follow the order of operations.

A number-based introduction to the distributive property has the additional strength of allowing students to confirm it—they can do the multiplication in a traditional manner as well as "in parts" to see if they are equivalent. We follow this introduction with two additional elaborations of the distributive property that involve algebraic expressions and rectangle area, both of which build on ideas developed earlier in the course. The combined effect of three mutually-reinforcing depictions helps to solidify student understanding of this very important idea.

Appendix J – Discussing Subtraction as Practice in Scientific Thinking

Signed number arithmetic has not been mastered by many GED graduates who arrive in CTP. It is a challenge to improve student understanding around signed numbers when students have experienced very different pedagogical approaches to these concepts in prior classes. Students may have signed number rules echoing around in their heads but frequently misapply them. For example, “positive and positive is a negative” is a mantra that students may misuse when adding two negative numbers. Put simply, memorizing signed number rules has not worked well for many of our students.[69]

Rather than presenting students with mathematical relationships that we define for them, CTP math instructors call on students to observe, reason, and talk about patterns and other relationships they notice on the way to discovering new ideas. When we use inductive activities that require students to make sense of and articulate mathematical relationships that are in front of them, they are getting practice in thinking and talking like scientists do. An example of this occurs very early in CTP when we investigate subtraction of signed numbers.

As an introduction to integer subtraction, we begin by facilitating a conversation about a subtraction problem such as $8-5$ because, while many students can “do” the problem, they may have difficulty in saying whether they are subtracting 8 or 5. It actually helps for us to reach back to more informal vocabulary. Many of us used “take away” to describe subtraction when we were children. In the expression $8-5$, we can ask students *“What am I taking away?”* as well as, *“What am I starting with?”*

The critical component of this lesson is in guiding the class to recognize and articulate the relationship between addition and subtraction. Instead of stating this relationship, we put the following addition and subtraction problems on the board for students to complete:

$8-5=$	$10-1=$	$9-2.5=$
$8+(-5)=$	$10+(-1)=$	$9+(-2.5)=$

All students can do the calculations at this stage in CTP because the subtraction problems do not draw them into negative territory, and because students are capable of adding signed numbers as a result of the previous lesson.

$8-5=3$	$10-1=9$	$9-2.5=6.5$
$8+(-5)=3$	$10+(-1)=9$	$9+(-2.5)=6.5$

Once these results have been articulated and confirmed by the students, it is time for a challenging conversation. Basically, we ask the students *“What do you see? What is going on here?”* CTP instructors need to be highly skilled to guide this discussion because at this early stage in the course, students are still new to our approach to communication and they rarely use precise language. The most frequent student response is: *“They’re the same.”* We have to press students to use language that describes exactly what is the same and what is different about the equation pairs.

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[69] Signed number rules can be presented with a formality that itself can be an obstacle to student understanding. See the following rule printed in a college remedial algebra textbook: “To add two integers with unlike signs, subtract their absolute values (the smaller from the larger) and append the sign of the integer with larger absolute value. Example: $4+(-8)=-(8-4)=-4$”

Eventually, a number of observations will be articulated by students, but often not without a lot of pushing for clarification and confirmation by the instructor. Eventually, most students will agree that for each of the three equation pairs, we are "starting" and "ending" with the same quantity. *"But what is happening in the middle?"* The operations are not the same and the numbers are not the same. Because we know that we started and ended in the same place, though, what happens in the middle must be the same in some way.

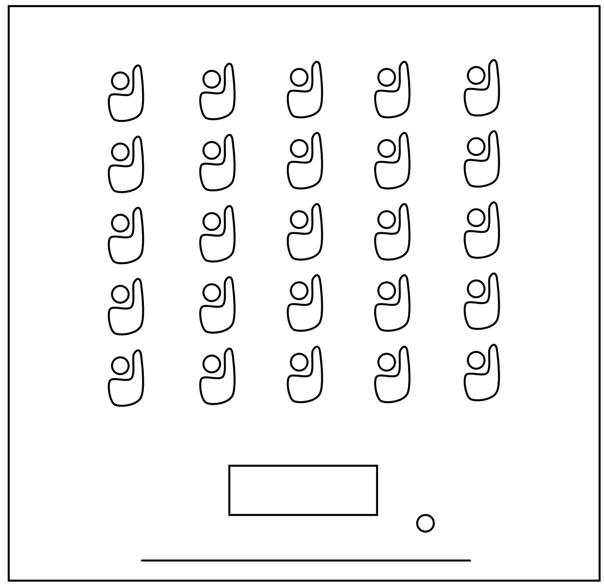
This conversation will arrive at an elaboration of a critical mathematical relationship—that *subtracting a number is the same as adding its opposite*. I do not want to give the impression that this conclusion comes easily. All CTP instructors have found this to be one of the most challenging conversations we have with our students. In a few cases, students can show frustration with our insistence on clear, accurate statements. Still, these kinds of conversations will occur on a regular basis over the semester and students do improve in their ability to articulate their reasoning, to make mathematical connections, and to be patient enough to persist in what can be some very technical conversations. Our larger goal is to structure this sort of thinking and these conversations often enough that students develop a scientist's habit of looking for, describing, and questioning the patterns they see all around them.

Appendix K – The Physical Environment Impacts Student Talk

The physical classroom environment can affect student communication and an instructor's ability to assess student work. *Figure 1* and *Figure 2* are examples of typical classroom orientations while *Figure 3* and *Figure 4* are examples of classroom orientations we encourage in CTP math classrooms.

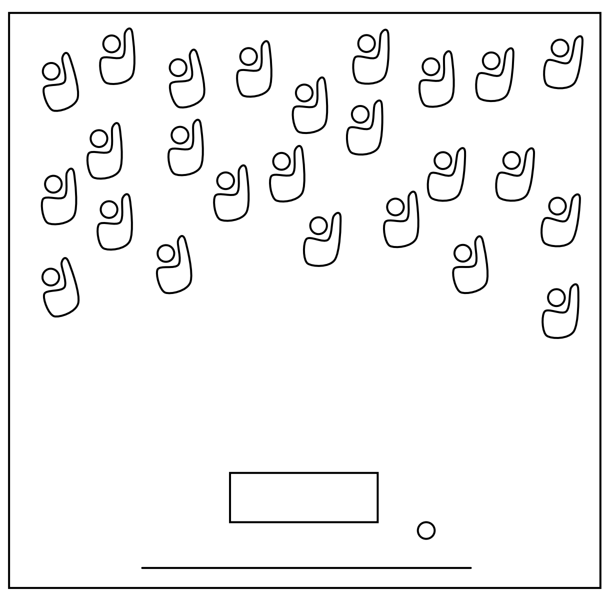
This bird's-eye view of a typical classroom includes individual student desks in orderly rows that face the instructor and board. It can be difficult for students to hear and address one another because they only have direct eye contact with the instructor. This format certainly signals that the important things happen at the front of the room.

Figure 1



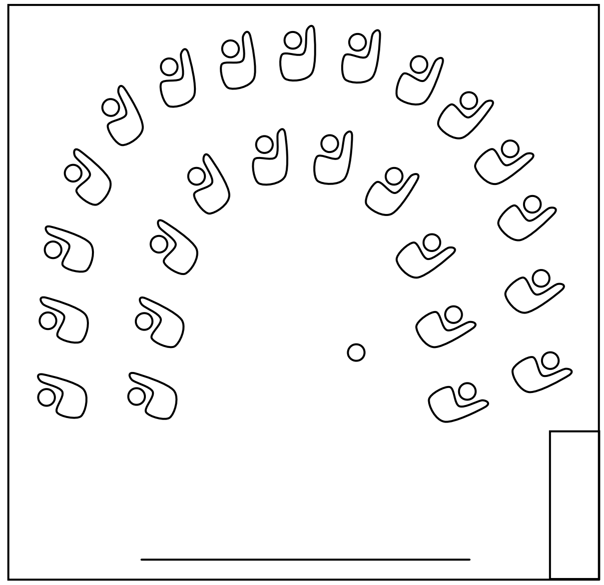
While student desks may begin in orderly rows, over the course of the day or week students can shift them into this orientation. Student desks continue to face the board and instructor rather than each other. The distance between the instructor and students has grown. It is now more difficult for the instructor to wander and engage students directly with their written work because access to the rear desks is blocked.

Figure 2



Moving desks into a double-U shape (shown in *Figure 3*) has several advantages over the seating shown in *Figures 1* and *2*. The overall orientation signals that students and instructor are all important parts of the classroom. Students can see, hear, and respond to one another which can facilitate the types of student talk we wish to foster in CTP. The instructor can also move quickly around the half-circles in order to engage with students' work. No student is really sitting in "the back". Desks are evenly spaced out in the room so that students can be easily moved into groups for an activity.

Figure 3



Steve and Christina work with students around the "double-U".

Photo by Sam Seifnourian

Tables can be arranged in a number of ways that achieve the same effects as in *Figure 3*. Students can easily see and talk to each other and the instructor, group work opportunities are obvious, and the instructor can easily move around the classroom to observe student work.

Figure 4